## A MESH FREE METHOD FOR THE NUMERICAL SOLUTION OF 1D NON-LINEAR UNDAMPED SINE-GORDON EQUATION

**Samreen Abbas**
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Joint work with: Tahira Nasreen Buttar

A simple collocation pseudo spectral method is formulated for the numerical solution of undamped Sine-Gordon (SG) equation using collocation points and approximated the solution using Thin Plate Spline (TPS) radial basis function. The results of numerical experiments are also presented to confirm the accuracy of the method.

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## EDGE IRREGULAR TOTAL LABELING OF GRAPHS

**Ali Ahmad**
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Joint work with: Martin Baca

An edge irregular total $k$-labeling of a graph $G = (V, E)$ is a labeling $f : V \cup E \to \{1, 2, \ldots, k\}$ such that the total edge-weights $wt(xy) = f(x) + f(xy) + f(y)$ are different for all pairs of distinct edges. The minimum $k$ for which the graph $G$ has an edge irregular total $k$- labeling is called the total edge irregularity strength of $G$.

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## STABILIZATION OF KAPITZA OSCILLATOR BY PERIODIC SYMMETRIC PULSES

**Babar Ahmad**
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*Islamabad-Pakistan.*

With the application of modified Kapitza procedure of averaging, we stabilize the oscillator moving under the influence of periodic external forces. We achieve this control by minimizing effective potential energy function. Our aim is to low down the frequency of oscillation as compared to harmonic/ periodic kicking pulses.

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## COHENMACAULAY INTERSECTIONS

**Safyan Ahmad**
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For a subset $F \subset [n]$, let $P_F$ be the prime ideal generated by the $x_i$ with $i \in F$. The minimal prime ideals of a squarefree monomial ideal $I$ in the polynomial ring $S = K[x_1, ..., x_n]$ over a field $K$ are all of this form, and since $I$ is radical ideal it is the intersection of its minimal prime ideals, say, $I = \bigcap_{i=1}^{r} P_{F_i}$ with $F_i \subset [n]$. Suppose $I$ is Cohen-Macaulay. For which exponents $a_{ij}$ is the ideal $J = \bigcup_{i=1}^{r}(x_j^{a_{ij}} j \in F_i)$ again Cohen-Macaulay? Here we restrict our attention to subsets $F \subset [n]$ with $|F| = 2$.

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## MULTICOMPLEXES AND POLARIZATION

**Sarfraz Ahmad**
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*Lahore-Pakistan.*

We define nice partition of the multicomplex associated to a Stanley ideal. In the main result we show that if the multicomplex associated to monomial ideal $I$ has a nice partition then the multicomplex associated to polarized ideal $I^p$ has a nice partition.

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## POINCARE SERIES OF MILNOR ALGEBRAS AND FREE ARRANGEMENTS

**Shahid Ahmad**
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*Lahore-Pakistan.*

We show that for a free complex hyperplane arrangement $A : f$, the Poincare series of the graded Milnor algebra $M(f)$ and the Betti numbers of the arrangement complement $M(A)$ determine each other. Examples show that this is false if we drop the freeness assumption.

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## CREMONA TRANSFORMATIONS AND MILNOR ALGEBRAS

**Imran Ahmed**
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*Lahore-Pakistan.*

In Theorem 3.2 we show that two homogeneous polynomials $f$ and $g$ having isomorphic Milnor algebras are right-equivalent. This is similar to the celebrated theorem by Mather and Yau[6], saying that the isolated hypersurface singularities are determined by their Tjurina algebras. Our result applies only to homogeneous polynomials, but it is no longer necessary to impose the condition of having isolated singularities at the origin.

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## REITERATION FOR THE $K$-INTERPOLATION METHOD IN LIMITING CASES

**Irshaad Ahmed**
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Joint work with: D. E. Edmunds, W. D. Evans, G. E. Karadzhov

Sharp reiteration theorems for the $K$-interpolation method in lim- iting cases are proved using two-sided estimates of the $K$-functional. As an application, sharp mapping properties of the Riesz potential are derived in a limiting case. Namely, let $(A_0, A_1)$ be a compatible pair of quasinormed spaces, and for each $f \in A_0 + A_1$ let

$$K(t, f) = K(t, f; A_0, A_1) = inf\{\|f_0|A_0\| + t\|f_1|A_1\|\}, t > 0, f = f_0 + f_1,$$

be the $K$-functional of Peetre. Consider the weighted Lebesgue space $L_q(v)$ defined by the quasi-norm

$$\|g|L_q(v)\| := \begin{cases} (\int_0^\infty v^q(t)|g(t)|^q \frac{dt}{t})^{1/q}, & 0 < q < \infty, \\ \sup\limits_{0<t<\infty} \{v(t)|g(t)|\}, & q = \infty \end{cases}$$

where $v$ is a positive, locally integrable function on $(0, \infty)$. Define $A_{v,q}$ with the norm $\|f|A_{v,q}\| = \|K(t, f)|L_q(v)\|$.
We show that a variant of Holmstedt's argument works for general weights. For example, we have for $W := \rho_0 + h_0$,

$$K(W, f; A_{w_0,p_0}, A_0) \approx (\int_0^t w_0^{p_0}(u) K^{p_0}(u, f) \frac{du}{u})^{(1/p_0)} + \frac{\rho_0(t)}{t} K(t, f)$$

where

$$\rho_0(t) = t(\int_t^\infty w_o^{p_0}(u) \frac{du}{u})^{1/p_0}, h_0(t) = (\int_0^t u^{p_0} w_0^{p_0}(u) \frac{du}{u})^{1/p_0}$$

A similar formula is proved for the couple $(A_0, A_{w_1,p_1})$.

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# EXACT SOLUTIONS FOR THE AXIAL COUETTE FLOW OF A GENERALIZED MAXWELL FLUID INDUCED BY TIME DEPENDENT SHEAR STRESS


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The axial Couette flow of a generalized Maxwell fluid is here discussed. The velocity field and the shear stress corresponding to the flow in an infinite circular cylinder are obtained by means of the Laplace and Hankel transforms. The motion is caused by the infinite cylinder which is under the action of a longitudinal time dependent shear stress. Both solutions are written in terms of the generalized $G$-functions. The similar solutions for ordinary Maxwell and Newtonian fluids are obtained as limiting cases from our general solutions for $\alpha \to 1$ and $\alpha \to 1, \lambda \to 0$ respectively. Finally, the influence of $\lambda$ and the fractional parameter $\alpha$ on the velocity field and shear stress is also analyzed by graphical illustrations.

## ON LINEAR CONTROLLABILITY

**M. Saeed Akram**
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A new defenition of controllability for linear dynamical systems is given. A linear system is said to be controllable if every its trajectory is achieved with finitely many differentiations from a trajectory having zero initial condition. It is shown then that the property of controllability is equivalent to the property of having "moving average" representation.

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## SUPER VERTEX-ANTIMAGIC TOTAL LABELINGS OF DISCONNECTED GRAPHS

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Joint work with: Martin Baca, Yuqing Lin, Andrea Semanicova-Fenovcikova

Let $G = (V, E)$ be a finite non-empty $(p, q)$-graph of order $p$ and size $q$ . An $(a, d)$-vertex-antimagic total labeling is a bijection $f$ from $V(G) \cup E(G)$ onto the set of consecutive integers $1, 2, \ldots, p + q$, such that the vertex-weights form an arithmetic progression with the initial term $a$ and the common difference $d$ , where the vertex-weight of $x$ is the sum of values $f(xy)$ assigned to all edges $xy$ incident to vertex $x$ together with the value assigned to $x$ itself, i.e. $f(x)$. Such a labeling is called super if the smallest possible labels appear on the vertices.
In this paper, we will study the properties of such labelings and examine their existence for disconnected graphs.

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## ON PATH-SUNFLOWER RAMSEY NUMBERS

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Joint work with: I. Tomescu

For given graphs $G$ and $H$, the Ramsey number $R(G, H)$ is the least natural number $n$ such that for every graph $F$ of order $n$ the following condition holds: either $F$ contains $G$ or the complement of $F$ contains $H$. In this paper, we determine the Ramsey number of path $P_n$ versus sunflower graph $SF_m$ when $n$ is at least quadratic relatively to $m$. In this case $R(P_n, SF_m) = 3n - 2$ if $m$ is odd and $2n + \frac{m}{2} - 2$ otherwise.
**Keywords:** Ramsey number, path, sunflower graph.

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## GRÖBNER BASES OF BRAID MONOIDS AND NORMAL FORMS OF BRAIDS

**Usman Ali**
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*Lahore-Pakistan.*

In this paper we find reduced Gröbner bases of braid monoids and obtain canonical form of a braid. We give a procedure to obtain an arbitrary divisor of Garside braid. We give a new method to compute initial set of these divisors. We also introduce a new way of finding exponent of a positive word. The results of this paper are used to improve the existing algorithms to find Garside normal form and left-greedy form of a braid. These normal forms are key ingredients in the solutions of word and conjugacy problem in braid groups.

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## CAUCHY MEANS OF THE LEVINSON TYPE AND RELATED RESULTS

**Matloob Anwar**
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*Lahore-Pakistan.*

Joint work with: Josip Pečarić

In this paper we give an improvement and a reversion of the well known Ky-Fan inequality as well as some related results. In this paper we introduce Levinson means of Cauchy's type. We show that these means are monotonic.

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## RECURRENCE RELATIONS FOR ALEXANDER CONWAY POLYNOMIAL

**Rehana Ashraf**
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*Lahore-Pakistan.*

A new method of computing Alexander Conway polynomial is introduced. It is developed by using simple recurrence relations. This method is simple and faster than the classical methods : Grassner Burau representation, using free differential calculus, or Siefert matrix.
**Keywords:**Skein relation, Recurrence relation

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## BOUNDEDNESS AND COMPACTNESS CRITERIA FOR A CLASS OF INTEGRAL OPERATORS WITH POSITIVE KERNELS

**Muhammad Asif**
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*Lahore-Pakistan.*

Joint work with: Usman Ashraf, Alexander Meskhi

Necessary and sufficient conditions governing the boundedness/compactness of positive kernel integral operators defined on cones of homogeneous groups are derived. Two-sided estimates of the measure of non-compactness and singular numbers for these operators are established.

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## FLOW OF A GENERALIZED MAXWELL FLUID INDUCED BY A CONSTANTLY ACCELERATING PLATE BETWEEN TWO SIDE WALLS

**Muhammad Athar**
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*Lahore-Pakistan.*

The unsteady flow of a viscoelastic fluid with the fractional Maxwell model, induced by a constantly accelerating plate between two side walls perpendicular to the plate, is investigated by means of the integral transforms. Exact solutions for the velocity field are presented under integral and series forms in terms of the derivatives of generalized Mittag.Leffler functions. The corresponding solutions for Maxwell fluids are obtained as limiting cases for $\beta \to 1$. In the absence of the side walls, all solutions that have been determined reduce to those corresponding to the motion over an infinite plate.

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## APPROXIMATION OF PSEUDORESOLVENT

**Abdus Sami Awan**
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This paper is devoted to approximation of pseudoresolvents and their generators. If $R_n, R : \Lambda \to L(X), n \geq 1$ are generated pseudoresolvents and $A_n$, $A$ their generators, we investigate conditions under which $A$ is approximated by $A_n$ and $R$ is approximated by $R_n, n \geq 1$. In addition, we give conditions under which a sequence of generated pseudoresolvents approximates a pseudoresolvent, and in this case we study the connection between generators.

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## STUDENTS' VIEWS OF MATHEMATICS: A SURVEY OF SOME GHANAIAN SCHOOLS

**Ernest K. Awanta**
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*University of Education,*
*Winneba-Ghana.*

This study investigated the views of students, their conceptions of mathematics, attitude toward and habits of learning mathematics, and their perceived difficulty level of various mathematics topics in Ghana. First, a convenience sampling procedure was adopted in sampling 10 junior high schools and 15 senior

high schools Second, a random sample was used to select 800 junior high and senior high students, to whom a questionnaire survey was administered. The data collected showed a clear picture of students' perception of mathematics learning with regard to categories of interest, preference for understanding, confidence and competence, textbooks, classroom learning and outside-class learning, and learning habits. It also depicted substantial trends of changing views and attitudes toward mathematics learning across grade levels. Students' responses to the Conception of Mathematics were consistent with previous studies, and demonstrated some specific characteristics of their views of mathematics. This survey has provided useful background information regarding students' needs and aspirations in mathematics learning for curriculum planners and frontline teachers in future curriculum reform and implementation.
**Key words:** Junior High School, Senior High School, Perception of Mathematics Learning, Conception of Mathematics, Trends of Students' Attitudes, Attitudes Toward Mathematics.

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## STRONG SUPER EDGE-MAGIC LABELINGS OF FORESTS

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Joint work with: Y. Lin, F. A. Muntaner-Batle, M. Rius-Font

A labeling of a graph is any map that carries some set of graph elements to numbers (usually to the positive integers). Magic labelings are one-to-one maps onto the appropriate set of consecutive integers starting from 1, with some kind of sum property. An edge-magic labeling on a graph with $p$ vertices and $q$ edges is defined as a one-to-one map taking the vertices and edges onto the integers $1, 2, ..., p+q$ with the property that the sum of the label on an edge and the labels of its endpoints is constant independent of the choice of edge. Such a labeling is called super if the smallest possible labels appear on the vertices.
In the talk, we introduce the concept of strong super edge-magic labeling as a particular class of super edge-magic labelings. The main goal is to use the strong super edge-magic labelings in order to show that the number of super edge-magic labelings of an odd union of path-like trees $mT$, all of them of the same order, grows very fast with $m$.

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## ON SUPER EDGE-MAGIC DEFICIENCIES OF FORESTS

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*Lahore-Pakistan.*

Joint work with: E.T. Baskoro, R. Simanjuntak, Ali Ahmad

Let $G = (V, E)$ be a finite, simple and undirected graph having $|V(G)| = p$ and $|E(G)| = q$. A super edge-magic labeling of a graph $G$ is a bijection $f : V(G) \cup E(G) \to \{1, 2, ..., p + q\}$, where $f(V(G)) = \{1, 2, \ldots, p\}$ and there exist a constant c such that $f(u) + f(uv) + f(v) = c$, for every edge $uv \in E(G)$. The super edge-magic deficiency of a graph $G$, denoted by $\mu_s(G)$, is the minimum nonnegative integer $n$ such that $G \cup nK_1$ has a super edge-magic total labeling or $+\infty$ if there exists no such $n$. In this paper, we study the super edge-magic deficiencies of a forest consisting at most three components. In particular, we determine the super edge-magic deficiency of a forest formed by paths, stars, comb, banana trees, and subdivision of $K_{1,3}$.

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## ON FACE ANTIMAGIC LABELINGS OF DISJOINT UNION OF PRISMS

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Joint work with: Gohar Ali, Martin Baca, Andrea Semanicova-Fenovcikova

We deal with the problem of labeling the vertices, edges and faces of a plane graph in such a way that the label of a face and the labels of the vertices and edges surrounding that face add up to a weight of that face and the weights of all $s$-sided faces constitute an arithmetic progression of difference $d$.
In the talk, we describe various antimagic labelings for the disjoint union of prisms.

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## ON CERTAIN MATRIX LIE RINGS OVER COMMUTATIVE RINGS

**Evgenii L. Bashkirov**
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Let $R$ be a commutative and associative ring, $n$ a positive integer, $M_n(R)$ the ring of all matrices of degree $n$ over $R$. If $g \in M_n(R)$, then ${}^tg$ is the transpose of $g$. Denote by $so_n(R)$ the set of all matrices $g \in M_n(R)$ such that ${}^tg - g$ and the trace of $g$ is equal to zero. Then $so_n(R)$ is closed under addition in $M_n(R)$ and under the Lie multiplication $[ab]ab - ba(a, b \in M_n(R))$, that is, $so_n(R)$ is a Lie ring with respect to these two operations. We prove the following result.
**Theorem.** Let $K$ be an associative and commutative ring with 1 and $k$ a subring of $K$ such that $1 \in k$. Assume that the elements 2 and 3 are invertible in $K$. Let $n$ be an integer, $n \geq 3, n \neq 4$. If $so_n(k) \subseteq \mathfrak{g} \subseteq so_n(K)$ and $\mathfrak{g}$ is a subring of the Lie ring $so_n(K)$, then $\mathfrak{g}so_n(A)$, where $A$ is a subring of $K$ containing $k$. This theorem is a continuation of studying matrix Lie rings started by the author in [1].

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## ON SOME RAMSEY-MINIMAL GRAPHS

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The notation of $F \to (G, H)$ means that in any 2-coloring on the edges of graph $F$ there exists a monocolor $G$ or $H$ in $F$. Let $\mathcal{R}(G, H) := \{F | F \to (G, H)$ and $F^* \not\to (G, H)$ for any proper subgraph $F^* \subset F\}$. The graph $F \in \mathcal{R}(G, H)$ is called a Ramsey $(G, H)$-minimal graph. It is interesting to find all graphs in $\mathcal{R}(G, H)$ for a pair of $G$ and $H$.
In this talk, we shall present some new results on the finding all Ramsey minimal graphs for some combinations of $G$ and $H$. In particular, we give some necessary and sufficient conditions for graphs belong to $\mathcal{R}(2K_2, H)$ and $\mathcal{R}(K_{1,2}, H)$ for some graph $H$.

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## A NUMERICAL METHOD FOR THE SOLUTION OF LINEAR SECOND ORDER BOUNDARY VALUE PROBLEM

**Farzana Begum**
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A numerical method for solving the second order linear boundary value problems is presented. The method is tested on twenty B.V.P's and it is found that numerical and exact solutions are in good agreement.

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## DIFFERENTIAL GEOMETRY WITH ITS APPLICATIONS ON 3D FEATURE LINES EXTRACTION

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Joint work with: Iman Yi Liao, Pan Zheng

Differential Geometry (DG) has found its applications in many areas. For example, in this paper the area we focus on is of extracting feature lines from 3D images, such as Computed Tomography (CT) data, and from 3D meshes generated from dense point clouds of an object surface obtained from a 3D scanner. Feature lines, such as crest lines (defined as the connection of maximum curvature points), are very useful information for registration of a subject to its reference model, which has been used for surgical purpose with CT data. Complete crest lines of an object surface are also able to recover the surface on their own, hence show the capability of representing a surface at very low cost. In this paper, we show how DG has been used to extract crest lines from both 3D images and 3D meshes. As for 3D images, an example of skull CT data is concerned. Following the Marching Lines algorithm proposed by J. P. [], we reinforce the concept and derivation of using only the second derivatives of a 3D image to calculate curvatures and by marking out the intersection of two iso-surfaces we obtain the crest lines. A Gaussian scale space modification to the ML algorithm is further proposed. Again we show how DG is combined with scale space operator to select features of interest and to deal with noisy data. We also implement an algorithm of extracting crest lines from a mesh of a skull surface that is generated by Marching cube method. We compare two methods and discuss the issues of accuracy related to the 3D images and meshes respectively.

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# UNIFIED FORMULAS FOR THE $n$th DERIVATIVE AND THE $n$th ANTI-DERIVATIVE OF ELEMENTARY AND SPECIAL FUNCTIONS

**Mhenni M. Benghorbal**
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A complete solution to the problem of finding the $n$th derivative and the $n$th anti-derivative of elementary and special functions will be given. In general, the solution is given through unified formulas in terms of the Fox $H$-function and the Miejer $G$-function which in many cases can be simplified to less general functions. One of the key points in this work is that the approach does not depend on integration techniques. The $n$th order of differentiation is found according to the Riemann-Liouville definition

$$f^{(n)}(x) = \frac{1}{\Gamma(k-n)} \frac{d^k}{dx^k} \int_a^x (x-t)^{k-n-1} f(t) dt, \;\; (k-1 < n < k),$$

where $k = \lceil n \rceil$, whereas the generalized Cauchy $n$-fold integral is adopted for the $n$th order of integration

$$f^{(-n)}(x) = \frac{1}{\Gamma(n)} \int_a^x (x-t)^{n-1} f(t) dt, \;\; n > 0.$$

The motivation of this work comes from the area of classical and fractional calculus as well as from the area of symbolic computation. The idea is that: given a function (elementary or special), can we or Computer Algebra Systems (CAS) find a formula for the nth derivative, the nth anti-derivative, or both? This enhances the power of integration and differentiation of CAS. A software exhibit will be within the talk using the computer algebra system Maple.

**Example 1:** A unified formula for $x^{1/4} arcsinh(\sqrt{x})$ in terms of the Meijer $G$-function

$$(x^{1/4} arcsinh(\sqrt{x}))^{(n)} = \frac{x^{(\frac{3}{4}-n)}}{2\sqrt{\pi}} G^{1,3}_{3,3}\left( \begin{array}{c} -\frac{3}{4}, \frac{1}{2}, \frac{1}{2} \\ 0, -\frac{1}{2}, n - \frac{3}{4} \end{array} \middle| x \right), \;\; |x| < 1$$

**Example 2:** A unified formula for the Guassian Hypergeometric Function $F\left( \begin{array}{c} a, b \\ c \end{array} \middle| x^2 \right)$ in term of the Meijer $G$-function

$$\left( F\left( \begin{array}{c} a, b \\ c \end{array} \middle| x^2 \right) \right)^{(n)} = \frac{2^n \Gamma(c) x^{(-n)}}{\Gamma(a)\Gamma(b)} G^{1,4}_{4,4}\left( \begin{array}{c} 0, \frac{1}{2}, 1-a, 1-b \\ 0, \frac{n+1}{2}, \frac{n}{2}, 1-c \end{array} \middle| -x^2 \right), \;\; |x| < 1$$

The above $G$-functions reduce to the original functions if $n = 0$. They give derivatives of any order if $n > 0$ and anti-derivative of any order if $n < 0$.

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## COXETER AND ARTIN GROUPS: GEOMETRY AND COMBINATORICS


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GARSIDE'S elements are related to the centers of the infinite families of Artin groups, the longest elements in Coxeter groups and galleries in the associated buildings. Graph-conbinatorial characterization and description of these elements are given, using relative Garside elements.


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## HOMOTOPY PERTURBATION METHOD FOR GINZBURG-LANDAU EQUATION


**Jafar Biazar**

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Joint work with: Masoumeh Partovi


In this paper we will consider the Ginzburg-Landau equation in the following form

$$iu_t + \alpha u_{xx} + \beta|u|^2u - bu - iau = 0.$$

Where $u$ is a complex valued function. This equation is originally discovered by Ginzburg and landau, for a phase transition in superconductivity. And context of the applications of the equation has been extended to various fields such as fluid mechanics, chemical reactions and pattern formation. Homotopy perturbation method (HPM) introduced by J. He in 1998, has been employed to derive an analytic approximation to the solution of Ginzburg-Landau equation, and results will be compared with those of Adomian decomposition method. To illustrate the ability and reliability of the method some examples are provided. The results

reveal that the method is very effective and simple.
**Keywords:** Ginzburg-Landau equation, Homotopy perturbation method.

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## ON THE CHROMATICITY OF MULTI-BRIDGE HYPERGRAPHS

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Joint work with: Ioan Tomescu, Akhlaq Ahmad Bhatti

A multi-bridge hypergraph is an h-uniform linear hypergraph consisting of some linear paths having common extremities. In this paper it is proved that the multi-sets of path lengths of two chromatically equivalent multi-bridge hypergraphs are equal provided the multiplicities of path lengths are bounded above by $2^{h-1}-2$. Also, it is shown that $h$-uniform linear cycles of length $m$ are not chromatically unique for every $m, h \geq 3$.
**Mathematics Subject Classification:** 05C15, 05C65

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## CHAOTIZATION BY PERIODICAL KICKING PULSES

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We study the two-dimensional dynamical system chaotization by periodical kicked pulses, when the smooth dynamical equations can be presented in the form of 1d or 2d discrete maps. For one-dimensional map we can apply the standard test with Schwarzian, and we discuss some ways of its multi-dimensional generalization.

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## SECANT LOCI OF RATIONAL NORMAL SCROLLS AND PROJECTIVE VARIETIES OF ALMOST MINIMAL DEGREE

**Markus Brodmann**
*University Of Zürich*
*Zürich-Switzerland.*

Joint work with: E. Park

Let $\tilde{X} = S(a_1, \ldots, a_n)$ be a standard rational normal scroll of codimension $> 1$ in projective $(r+1)-$space over an algebraically closed field. Keep in mind, that these scrolls together with all hyperquadrics and all cones over the Veronese surface form the class of projective varieties of minimal degree.
Let $p$ be a point in the ambient $(r+1)-$space $\mathbb{P}^{r+1}$ of $\tilde{X}$ but outside $\tilde{X}$ and let $Sec_p(\tilde{X})$ denote the secant cone of $\tilde{X}$ with respect to $p$, that is the closure of the union of $p$ with all secant or tangent lines to $\tilde{X}$ which run through $p$. Let $\sum_p(\tilde{X})$ be the secant locus of $\tilde{X}$ with respect to $p$, that is the intersection of $\tilde{X}$ and the secant cone $Sec_p(\tilde{X})$. Finally, let $X_p$ be the image of $\tilde{X}$ under a projection with center $p$ of our ambient $(r+1)-$space $\mathbb{P}^{r+1}$ to a projective $r$-space $\mathbb{R}^r$. Then $X_p$ is birationally equivalent to $\tilde{X}$ and a surface of almost minimal degree in its ambient $r$-space $\mathbb{P}^r$. Let $t$ be the arithmetic depth of $X_p$.
It turns out, that $Sec_p(\tilde{X})$ is a $(t-1)$-dimensional projective subspace of the ambient $(r+1)-$space $\mathbb{P}^{r+1}$ and $\sum_p(\tilde{X})$ is a hyperquadric in $Sec_p(\tilde{X})$. If $\tilde{X}$ is smooth only the following six cases occur
(1) $t = 1$ and $\sum_p(\tilde{X})$ is empty;
(2) $t = 2$ and $\sum_p(\tilde{X})$ consists of two distinct simple points:
(3) $t = 2$ and $\sum_p(\tilde{X})$ is a double point;
(4) $t = 3$ and $\sum_p(\tilde{X})$ consists of two distinct simple lines;
(5) $t = 3$ and $\sum_p(\tilde{X})$ is a smooth plane conic;
(6) $t = 4$ and $\sum_p(\tilde{X})$ is a smooth quadric surface.
In the general case, one correspondingly has 6 cases, although now the value of $t$ may exceed 4. Therefore, the ambient space $\mathbb{P}^{r+1}$ has a decomposition in 6 different strata, according to the position of the center $p$. We give a complete geometric description of these strata. In particular, we now can understand in terms of this secant stratification how the variety $X_p$ depends on the position of the center $p$.
We apply this to study varieties of almost minimal degree and notably get a complete classification of all non-normal Del Pezzo varieties. This extends Fujitas classification of normal Del Pezzo varieties. Another application concerns the embeddings scrolls $Y$ of $X_p$, for which we notably prove a unicity result.
**Key words and phrases:** Veronese surface, projective varieties.

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## POINCARÉ SERIES AND MULIINDEX FILTRATIONS

**Antonio Campillo**
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*Universidad de Valladolid,*
*Volladolid-Spain.*

Poincaré series involve enumerative and descriptive information on many mathematical phenomena. In joint work with F. Delgado and S. Gussein-Zade we have introduced a notion of Poincaré series for several variables, which, in cases of practical interest, it applies to some problems in combinatorics, arithmetics and geometry of singularities. We describe and review standard applications of Poincaré series to singularity theory, obtaining a relationship with zeta functions and Alexander polynomials in particular cases. We also describe and review recent new applications involving zeta functions and Alexander polynomials for ideals, the role of Rees valuations, and Poincaré series given by induced filtrations or filtrations of multiplier ideals.

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## ALGEBRA FOR COMPUTER SCIENCE

**Virgil Emil Căzănescu**
*Faculty of Mathematics,*
*University of Bucharest,*
*Bucharest-Romania.*

Every science tries to use mathematical tools to solve its problems. Com- puter Science has looked for such tools and a lot of them have been used successfully. But some branches of Computer Science did not find the necessary tools in the classical mathematics. Therefore, the computer scientists tried to introduce new mathematical concepts to model some phenomena in Computer Science. Our talk is on such a mathematical concept, an algebraic one.
Our algebraic structure basic network algebra is a strict monoidal category N having some transposition constants ${}^aX^b \in N(ab, ba)$ and a feedback operation $\uparrow^a$: $N(ab, ac) \to N(b, c)$ to model loops.
This algebraic structure has many applications in computer science and even in pure mathematics.
The applications in computer science regard the deterministic and nondeterministic programs and their semantics, the regular languages, the data-flow network, Petri nets, process algebra, etc.
As an example of application in mathematics, we mention the following theorem: The finite bijections form an intial basic network algebra.

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## CONSTRUCTING NUMERICAL CAMPEDELLI SURFACES

**Wolfram Decker**
*Fachrichtung Mathematik,*
*Universität des Saarlandes,*
*Saarbrücken-Germany.*

Through the power of modern computers and effective computer algebra methods, algebraic geometry has become accessible to experiments. In this lecture, I report on potential applications to the classical problem of describing families of surfaces of general type with small invariants.

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## FROM SCHREIER DOMAINS TO T-SCHREIER DOMAINS

**Tiberiu Dumitrescu**
*Faculty of Mathematics and Informatics,*
*University of Bucharest,*
*Bucharest-Romania*

Joint work with: Muhammad Zafrullah

P.M. Cohn [1968] introduced the concept of a Schreier domain: an integrally closed domain whose group of divisibility satisfies the Riesz interpolation property. Later on, extensions of this concept were given. In this talk, we present some moments in the evolution of the theory of Schreier domains. Also, we introduce and study a related concept, $t$-Schreier domain, a domain whose group of $t$-invertible $t$-ideals satisfies the Riesz interpolation property.

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## GENERALIZATION OF $K$-DIVERGENCE AND RELATED MEANS

**G. Farid**
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*Lahore-Pakistan.*

In this paper, we give generalization of $K$-divergence measure and related results by using some log-convexity criteria. We will also give new Cauchy means and prove monotonicity of these means.

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## DELVING THE PROBLEMS OF MATH TEACHING AT HIGHER SECONDARY SCHOOLS

**Muhammad Ajmal Farooq**
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Joint work with: Muhammad Safdar, Muhammad Zafar Iqbal, Irshad Baloch

Education plays an important role in building and moulding the character of an individual. It determines the status of nation among the community of nations. Education helps a person to understand himself and his environment in which he is living.
In dark ages, power ruled over the world but in the present age, science ruled over the world. So we can say that Math is free of time, space and nation. We are passing through the age of Math. The present progress in the world owes to Math. It has brought revolution in our daily life. Math is in fact protest against the problems of our daily life. We can say that the math is a national building activity and it will be play an increasingly important role in our future development.
We need the highest standard of Mathematics. In the existing system of Pakistan, Math education neither meets the requirements and nor fulfils the demands and basic needs of students and teachers. It requires overall changes. It is only possible if we find out drawbacks, shortcomings, and major problems in math education. Higher secondary schools are newly born. Lack of well-trained staff, library books are the major hurdles in higher secondary schools. Math teaching is facing many problems, and difficulties of varying nature. It is very difficult to change the whole set-up of Math education at once, but we can moderate this system stepwise.
The main objective of the study to study the factors influences the teaching of Mathematics.
For conducting the study, all Math teachers working in the male and female Higher Secondary Schools of Punjab Province and all male and female Math students. It is important to note that in Punjab Province the number of teachers in higher secondary schools is higher than the number of students.

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## DISCONNECTED AND REGULAR SUPER $(A, D)$-EDGE-ANTIMAGIC TOTAL GRAPHS

**Andrea Feňovčíková**
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*Slovak Republic.*

Joint work with: Martin Bača, Francesc Antoni Muntaner-Batle, Yuqing Lin, Muhammad Kashif Shafiq, Petr Kovář

A *labeling* of a graph is a mapping that carries some set of graph elements into numbers (usually the positive integers). An $(a, d)$-*edge-antimagic total labeling* of a graph $G(V, E)$ is a one-to-one mapping that takes the vertices and the edges

onto the integers $\{1, 2, \ldots, |V(G)| + |E(G)|\}$, such that the sums of the label on the edges and the labels of their end vertices form an arithmetic progression starting at $a$ and having a difference $d$. Such a labeling is called *super* if the smallest possible labels appear at the vertices.

In the first part of the contribution we give some new results on the super $(a, d)$-edge-antimagic total labelings of disconnected graphs. More precisely, we will deal with the differences $d \in \{0, 1, 2, 3\}$. In the second part we prove that every even regular graph and every odd regular graph with a 1-factor are super $(a, 1)$-edge-antimagic total. We also introduce some constructions of non-regular super $(a, 1)$-edge-antimagic total graphs.

**Keywords:** super $(a, d)$-edge-antimagic total labeling, disconnected graph, regular graph.

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## TAYLOR-COUETTE FLOW OF A GENERALIZED OLDROYD-B FLUID DUE TO A CIRCULAR CYLINDER SUBJECT TO A TIME-DEPENDENT COUPLE

**C. Fetecau**
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Joint work with: I. Burdujan, Corina Fetecau

This paper deals with a relevant problem in physics and engineering, namely the Taylor-Couette flow of an Oldroyd-B fluid in an infinite circular cylinder subject to a time-dependent shear stress. The main objectives are:
1. to establish exact solutions for the velocity field and the shear stress,
2. to present these solutions in suitable forms and
3. to correct some erroneous results that have been recently obtained for Newtonian fluids.

As a result of this endeavor, our solutions are presented as a sum of the Newtonian solutions and the corresponding non-Newtonian contributions. These solutions can be easy specialized to give both the solutions for generalized second grade and Maxwell fluids and the solutions for ordinary fluids performing the same motion. The corresponding Newtonian solutions are also obtained as a limiting case of the general solutions. Furthermore, the non- Newtonian contributions of general solutions are also presented in equivalent forms, under series form, in terms of the time derivatives of the Newtonian solution.

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## INTERSECTION THEOREMS FOR FINITE SETS

**Peter Frankl**
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*Paris-France.*

Let $[n]$ denote the set $\{1, 2, \ldots, n\}$ and $2^{[n]}$ the power set of $[n]$. A subset $F$ of $2^{[n]}$ is called a family. The basic problem of extremal set theory is to give upper and lower bounds on the zise, $|F|$ of $F$ supposing that satisfies some conditions. Natural examples are t. intersecting ($F \cap F'| \geq 1$ for all $F, F' \in F$), s-union ($|F \cup F'| \leq n - s$ for all $F, F' \in F$).
The maximum size of $t$-intersecting families is easily seen to be the same as that of $t$-union families. This maximum was determined by Katona in 1964. Following a conjecture of Katona, Frankl determined the maximum size of t-intersecting and 1-union families in 1975. However, the maximum size of families which are simultaneously s-union and $t$-intersecting with both $s$ and $t \geq 2$ is unknown even now. We shall survey this and related problems.

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## NUMBER OF SOLUTIONS OF POLYNOMIAL SYSTEMS OVER FINITE FIELDS

**Sudhir R. Ghorpade**
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*Indian Institute of Technology Bombay,*
*Mumbai-India.*

Joint work with: Sartaj Ul Hasan, Pradhan Prashant Kumar

Let $\mathbb{F}_q$ denote the finite field with $q$ elements and let $\mathbb{P}^m = \mathbb{P}^m(\mathbb{F}_q)$ denote the $m$-dimensional projective space over $\mathbb{F}_q$. We are interested in the following question.

What is the maximum number of common zeros in $\mathbb{P}^m$ that a system of $r$ linearly independent homogeneous polynomials, each of degree $d$ in $m+1$ variables with coefficients in $\mathbb{F}_q$, can have?

Let $N(r, m, d; q)$ denote this maximum number. When $d = 1$, this is a question of linear algebra and the answer is easy: $N(r, m, 1; q) = \pi_{m-r}$ where $\pi_j := |\mathbb{P}^j(\mathbb{F}_q)| = q^j + q^{j-1} + \ldots + q + 1$ for any nonnegative integer $j$. Next, suppose $r = 1$. Intuitively speaking, a hypersurface $H$ of degree $d$ in $\mathbb{P}^m$ projects onto $\mathbb{P}^{m-1}$ and above each point of $\mathbb{P}^{m-1}$, there are at most of $d$ points of $H$. Thus it seems natural to expect that $N(1, m, d; q) = d\pi_{m-1}$. However, it turns out that for $1 < d < q$, this is not true and it was conjectured by Tsfasman that $N(1, m, d; q) = dq^{m-1} + \pi^{m-2}$ for $d < q$. This has been proved, in the affirmative, by Serre [4], and independently, by Srrensen [5]. The general case of the above

question is still open and there is a rather elaborate conjecture of Tsfasman and Boguslavsky concerning $N(r, m, d; q)$. We shall present an elementary proof of this conjecture when $r = 2$, which can be viewed as a simplified version of the work of Boguslavsky [1]. The proof in the case $r = 2$ is based on a general inequality given in [2] for the number of points of projective algebraic sets.
We remark that the above question, as well as its analogue in the affine case, is intimately related to the theory of error correcting codes. More precisely, it corresponds to the determination of the $r$th higher weight of $q$-ary projective (as well as affine) Reed-Muller codes. Further, we remark that the analogous question in the affine case is completely settled in [3]. These connections and some of the related known results shall also be outlined.

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# KOSTKA FOULKES POLYNOMIALS AT CUBE ROOTS OF UNITY

**Ashar Ghulam**
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In this paper Kostka Foulkes Polynomials which are entries of transition matrix between Schur functions and Hall-Littlewood Symmetric functions are discussed at cube roots of unity. Schur functions and Hall-Littlewood Symmetric functions are bases of ring of all homogenous symmetric polynomials of degree $n$ in independent variables $x = (x_1, x_2, \ldots, x_n)$. Here we have derived three results which give relationship between $K_{\lambda\mu}(t)$ and different values of $\lambda$ and $\mu$ at cube roots of unity. Here $\lambda$ and $\mu$ are partitions of a positive integer $n$.

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## ON COMPLETION OF NONLINEAR DIFFERENTIAL SYSTEMS TO INVOLUTION

**Vladimir P. Gerdt**
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*Dubna-Russia.*

In this talk we consider some constructive aspects of algebraic completion of polynomially-nonlinear systems of partial differential equations (PDEs) to involution. To optimize the computational costs of prolongations and projections to be done in the course of completion one can use separation of independent variables for the equations in the system based on theory of involutive divisions [1]. To preserve the solution space by performing completion one has to make sure that the initials and separants of equations in the system do not vanish on its solutions. This condition is fulfilled by performing the triangular decomposition of the nonlinear differential system into algebraically simple subsystems. The notion of algebraically simple system was introduced by Thomas [2] who designed also the decomposition method modified in [3,4]. We discuss this method and show how algebraic Groebner bases can be used to avoid unnecessary splitting in the Thomas approach. Finally, we show how to combine the splitting method with the involutive completion technique [5] to decompose of a nonlinear PDE system into a finite set of involutive and algebraically simple subsystems with disjoint set of solutions.

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## OPTIMAL EMBEDDINGS OF BESSEL-POTENTIAL-TYPE SPACES INTO GENERALIZED HLDER SPACES INVOLVING K-MODULUS OF SMOOTHNESS

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*Prague-Czech Republic.*

We establish necessary and sufficient conditions for embeddings of Bessel potential spaces $H^\sigma X(\mathbb{R}^n)$ with order of smoothness $\sigma \in (0, n)$, modelled upon rearrangement invariant Banach function spaces $X(\mathbb{R}^n)$, into generalized Hölder spaces (involving $k$-modulus of smoothness). We apply our results to the case when $X(\mathbb{R}^n)$ is the Lorentz-Karamata space $L_{p,q;b}(\mathbb{R}^n)$. In particular, we are able to characterize optimal embeddings of Bessel potential spaces $H^\sigma L_{p,q;b}(\mathbb{R}^n)$ into generalized Hölder spaces. Applications cover both superlimiting and limiting cases. We also show that our results yield new and sharp embeddings of Sobolev-Orlicz spaces $W^{k+1}L^{n/k}(\log L)^\alpha(\mathbb{R}^n)$ and $W^kL^{n/k}(\log L)^\alpha(\mathbb{R}^n)$ into generalized Hölder spaces.

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## A REPRESENTATION OF A RING OF NEWTON INTERPOLATING SERIES

**Azeem Haider**
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Joint work with: S. M. Ali Khan

Let $K$ be a field and let $S$ be a sequence of elements of K. If $K_S[[X]]$ is the $K$-algebra of formal series called Newton interpolating series (see [1]), we study algebraic properties of this algebra and in the case when $S$ has a finite number of distinct elements we prove that it is isomorphic to a direct sum of a finite number of algebras which are either equal to $K[[X]]$ or to factor rings of $K[X]$.

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## A NOTE ON BOUNDS FOR TRAVELING SALESMAN PROBLEM


**Fozia Hanif Khan**

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The purpose of this research is to detect a mathematical error (Bounds for TSP by Nicos Christo 1975, 25 March) by providing the value of a TSP path which is less then the provided lower bound of the example proposed by the author.


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## ON ANALYTICAL FINDING PROBABILITY DISTRIBUTION LAW OF LINEAR COMBINATION OF EXPONENT QUARDATIC FORMS OF NORMALLY DISTRIBUTED RANDOM VECTORS


**M. A. Hashmi**

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*Lahore-Pakistan.*

Joint work with: K. J. Kachiashvili



In this paper there is considered the problem of analytical finding probability distribution law of linear combination of exponent of quadratic forms of normally distributed random vectors. This problem arises at solving different statistical problems, in particular, at testing many statistical hypotheses concerning parameters of normally distributed random vector. There is proofed that analytical finding this law is impossible when a number of quadratic forms is more than or equal to two, or, in particular, at testing statistical hypotheses, the number of hypotheses $S \geq 2$.


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## THE GEOMETRY OF ALGEBRAIC VARITIES AT THEIR SINGULAR POINTS

**Herwig Hauser**
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*Vienna-Austria.*

A point $p$ on a variety is singular, if the variety is not locally at $p$ isomorphic to a linear space. At these singular points, a whole world of geometric patterns emerges: cusps, edges, self-intersections, cones, cylinders, tangencies, ... The set of singular points forms an algebraic subvariety, which itself can be singular. If it is smooth, the behaviour of the variety along this locus may change continuously or show ruptures. If it is singular, stratifications are a convenient technique to describe the local geometry. The concept of blowups allows to zoom into the singular points. There thus evolves a kaleidoscope of classifications and descriptions of singularities.
In the talk, we will address the most common geometric features, illustrating them on visualizations of singular surfaces.

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## OPTIMAL QUADRATURE FORMULA WITH DERIVATIVE IN SOBOLEV SPACE $L_2^{(m)}(0,1)$

**A. R. Hayotov**
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*Tashkent-Uzbekistan.*

Joint work with: Kh. M. Shadimetov, F. A. Nuraliev

In the space $L_2^{(m)}(0,1)$ the problem of construction of optimal quadrature formulas is considered. Here the quadrature sum consists on values of integrand at nodes and values of first derivative of integrand at the end points of integration interval. The optimal coefficients are found and norm of the error functional is calculated for arbitrary fixed $N$ and for any $m \geq 2$. It is shown that when $m = 2$ and $m = 3$ the Euler-Maclaurin quadrature formula is optimal.

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## OPTIMAL QUADRATURE FORMULA WITH POSITIVE COEFFICIENTS IN SOBOLEV SPACE

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In the present work it is constructed the optimal quadrature formula of the form

$$\int_0^1 \varphi(x)dx \cong \sum_{\beta=0}^{N} C_\beta \varphi(x_\beta)$$

with the nodes

$$x_i = \eta_i h, \quad x_{N-i} = 1 - \eta_i h, \quad i = \overline{0, t-1}, \quad 0 \le \eta_0 < \eta_1 < ... < \eta_{t-1},$$

$$x_\beta = \beta h, \quad t \le \beta \le N - t, \quad h = \frac{1}{N}, N = 1, 2, ..., \quad t = \left[\frac{m}{2}\right]$$

in the $L_2^{(m)}(0,1)$ space. Choosing boundary nodes we obtained positive optimal coefficients.

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## HILBERT COEFFICIENTS AND POWERS OF IDEALS

**Juergen Herzog**
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*Univeristät Duisburg-Essen,*
*Essen-Germany.*

In this talk I report on work with Tony Puthenpurakal and Jugal Verma (to appear in Math.Proc.Cambr.Phil.Soc.), as well as on recent work with Dale Cutkosky and Hema Srinivasan. Let $K$ be a field, $S = K[x_1, \ldots, x_n]$ the polynomial ring in $n$ variables, and let $N$ be any graded $S$-module of dimension $d$. The Hilbert polynomial of $N$ is of the form $\sum_{i=0}^{d}(-1)^i e_i(N)\binom{x+d-i}{d-i}$. The integer coeffcients $e_i(N)$ are called the Hilbert coeffcients of $N$. It is shown that if $M$ is a graded $S$-module, and $I \subset S$ a graded ideal with $dim M/IM = d$. Then for all $i = 0, \ldots, d$, the Hilbert coefficient $e_i(M/I^kM)$ as a function of $k$ is a polynomial of degree $\le n - d + i$.
One may ask a similar question for $e_i(R/I^k)$, when $(R, m)$ is a Noetherian local ring and $I \subset R$ an ideal. Again $e_0(R/I^k)$ is a polynomial function for large $k$, but we do not know the answer for the other Hilbert coefficients. The problem is related to the question whether the algebra $A = \bigoplus_{k\ge 0}(I^k)^*$ is finitely generated. Here $J^*$ denotes the form ideal of $J$, that is, the ideal of leading forms of $J$ with respect to $m$ . Examples show that A not always finitely generated. We discuss, situations when $A$ is finitely generated. General criteria for finite generation of $A$ seem to be lacking. However if $I$ is an ideal in the power series ring $R = K[[x_1, \ldots, x_n]]$ one may attach to $I$ in a natural way an ideal $I^\sharp \subset R[[s]]$ with the property that $A$ is finitely generated if and only if the algebra $\bigoplus_{k\ge 0}(I^\sharp)^k : s^\infty$

of generalized symbolic powers of $I^\sharp$ is finitely generated. Criteria for finite generation of algebras of generalized symbolic Rees algebras are available.
Indeed, the study of this type of algebras has a long history starting with work of McAdam, Ratliff and others. We will discuss the rate of growth of such algebras.

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## POSITIVE DATA MODELING USING SPLINE FUNCTIONS

**Malik Zawwar Hussain**
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Joint work with: Asfar Nisar

A rational cubic function has been constructed to visualize the positive data that arises from some scientific phenomena. The main focus of the work is the representation of the data graphically in such a way that it looks smooth and attractive. In the first step simple data dependent constraints are derived on the free parameters in the description of the rational cubic function to visualize the shape of positive curve data then, it is extended to a rational bicubic partially blended functions (Coons-patches) and derived constraints on free parameters to visualize the shape of positive surface data. The developed scheme is local and computationally economical.

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## ERROR ESTIMATION OF RATIONAL CUBIC FUNCTION

**Maria Hussain**
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Joint work with: Malik Zawwar Hussain

The efficiency of different interpolating and approximating schemes is judged on the basis of computational time, memory usage, difficulty in implementation, error in computing, pleasance of graphical display and stability. The work in this paper is a study of error estimation of a $C^1$ rational cubic function with shape parameters using Peano Kernel Theorem. It is observed that the rational function under consideration is convergent but not symmetric about the free parameters. Accuracy of the curvature is also measured.

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## ON SUPER ANTIMAGIC TOTAL LABELING OF HARARY GRAPH

**M. Hussain**
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*Lahore-Pakistan.*

This paper deals with two types of graph labelings namely, the super $(a, d)$-edge antimagic total labeling and super $(a, d)$-vertex antimagic total labeling on the Harary graph $C_n^t$ . We also construct the super edge-antimagic and super vertex-antimagic total labelings for a disjoint union of $k$ identical copies of the Harary graph.
**Keywords:** Super antimagic total labeling, Harary graph.

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## BOUNDS FOR STRENGTHENED HARDY AND POLYA-KNOPP'S DIFFERENCES

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Joint work with: Josip Pečarić

In this paper we determine some Bounds for strengthened Hardy-Knopp differences, that is, an improvement and reverse of strength- ened Hardy-Knopp type inequality and its dual inequality is determined.

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## THE WEIGHTED SQUARE INTEGRAL INEQUALITIES FOR THE FIRST DERIVATIVE OF THE FUNCTION OF A REAL VARIABLE

**S. Hussain**
*Abdus Salam School of Mathematical Sciences,*
*Lahore-Pakistan.*

Joint work with: J. Pečarić, M. Shashiashvili

In this article we establish some weighted square integral estimates for the first derivative of a twice continuously differentiable function. We generalize the square integral estimate for the difference of derivatives of two arbitrary convex functions by Shashiashvili (2005) to the case of the family of the weight functions, satisfying certain conditions. This kind of generalization is especially valuable in the problems of mathematical finance for construction of the discrete time hedging strategies.

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## EXACT SOLUTION OF GOURSAT PROBLEMS USING DIFFERENTIAL TRANSFORM METHOD

**Muhammad Idrees**
*GIK Institute of Engineering Sciences and Technology, Swabi-Pakistan.*

Two-dimensional Differential Transform Method (DTM) is applied to derive closed form series solution of Goursat problems. Linear and nonlinear Goursat problems are considered to show fast convergence, simple applicability and efficiency of the new technique.

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## ON METRIC AND PARTITION DIMENSIONS OF SOME INFINITE REGULAR GRAPHS

**Muhammad Imran**
*Abdus Salam School of Mathematical Sciences, Lahore-Pakistan.*

In this paper some infinite regular graphs generated by tilings of the plane by regular triangles and hexagons are considered. These graphs have no finite metric bases but their partition dimension is finite and is evaluated in some cases. Also, it is proved that for every $n \geq 2$ there exists finite induced subgraphs of these graphs having metric dimension equal to n as well as infinite induced subgraphs with metric dimension equal to three.
**Keywords:** Metric dimension, partition dimension, plane tiling, infinite regular graph, induced subgraph.

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## A NEW MINIMUM ANGLE ALGORITHM FOR THE SOLUTION OF GENERAL LINEAR PROGRAMS

**Syed Inayatullah**
*University of Karachi,*
*Karachi-Pakistan.*

Joint work with: Nasiruddin Khan

In this paper we presented a new minimum angle rule of pivoting which is simple and comparatively much faster than the Dantzig's rule in solving systems of linear inequalities and linear programs. We also presented experimental results that compare the speed of our new rule to the classical methods. Although we hadn't proved the finiteness, the experimental results showed that this algorithm outperforms almost all the previous pivot rules including the Dantzig's simplex method.
**Keywords:** Linear programming, minimum angle algorithms, simplex method.

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## UNIQUE LIMIT CYCLE IN LIENARDS DYNAMICAL SYSTEM DRIVEN BY FAST CHANGING PERIODICAL FORCE

**Nayyer Iqbal**
*Abdus Salam School of Mathematical Sciences,*
*Lahore-Pakistan.*

We investigate the existence of a unique periodic orbit, or limit cycle in Lienards dynamical system (1928) driven by fast changing periodical external force, which actually modeled a vacuum tube that settled down to periodic output. Later it has been applied to many other phenomena. After the application of Kapitzas averaging procedure we formulate the conditions for the existence of unique limit cycle in the smooth coordinate system. A particular example of our method is given for Van der Pol equation.
**Keywords:** Lienards theorem, open-loop control.

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## UNIVERSAL UPPER BOUND FOR THE GROWTH OF ARTIN MONOIDS

**Zaffar Iqbal**
*Abdus Salam School of Mathematical Sciences,*
*Lahore-Pakistan.*

In this paper we study the growth rates of Artin monoids and we show that 4 is a universal upper bound. We also show that the generating functions of the associated right-angled Artin monoids are given by families of Chebyshev polynomials. Applications to Artin groups and positive braids are given.

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## VISUALIZATION OF DATA USING SPLINE FUNCTIONS

**Misbah Irshad**
*University of the Punjab,*
*Lahore-Pakistan.*

Visualization is a technique to convey a message through images, diagrams and animations. It has a lot of applications in engineering, medicince, geophsics, etc. In this paper, visualization of data that arises from some scientific phenoman or experiment is discussed. Simple data dependent constarints are derived on free parameters in the description of spline functions to visualize the shape of data.

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## A NEW METHOD FOR SOLUTION OF PHYSICAL PROBLEMS MODELED BY DIRAC DELTA FUNCTIONS

**Khalid Jamil**
*Department of Physics and Applied Mathematics,*
*Pakistan Institute of Engineering and Applied Sciences,*
*Islamabad-Pakistan.*

Customarily physics students of elementary level are not taught to model physical problems using Dirac delta function, simply because the differential equations involving Dirac delta function are not easily solvable by such a group of students. This paper presents an easy and elegant mathematical procedure to find solutions of ordinary differential equations, which arise in many of the physical problems, modeled by Dirac delta functions. Having knowledge of solving homogenous differential equation with given initial condition or boundary conditions is the only prerequisite to work with this new method. The method simply employs the concept of Heaviside function, and it derivative. The new method is enunciated for first and second order differential equations and can be generalized for higher order non-homogenous differential equation involving Dirac delta function.

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## ON VARIATIONS OF THE METRIC DIMENSION OF A GRAPH

**Imran Javaid**
*Bahauddin Zakariya University,*
*Multan-Pakistan.*

A subset $W = \{w_1, ..., w_k\}$ of vertices of a graph $G$ is called a resolving set for $G$ if for every two distinct vertices $x, y \in V(G)$ there is a vertex $w_i \in W$ such that $d(x, w_i) \neq d(y, w_i)$. A resolving set containing a minimum number of vertices is called a metric basis for G and the number of vertices in a metric basis is its metric dimension $\beta(G)$. In this paper, we study some variations of this parameter.

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## AVOIDING INFINITIES FROM THE LORENTZ AND THE POINCARÉ TRANSFORMATIONS

**Syed Arif Kamal**
*University of Karachi,*
*Karachi-Pakistan.*

From the historical point-of-view, this paper shall give history of conceptual development of relativity as well as mathematical formulation of Lorentz transformations. Contrary to the popular belief giving credit of all contributions in special relativity to Albert Einstein, the paper shall put into perspective fundamerntal conceptual contributions of Muslim scientists, e. g., space and time, relative and absolute, long before Einstein presented his theory. One year before the publication of Einstein's paper, Poincaré (1904) enunciated the principle of relativity. Also, complete mathematical framework was available through works of Voigt (1887) and Lorentz (1904). In his paper, 'Zur Electrodynamik bewegter Korper' (On the Electrodynamics of Moving Bodies), published in 1905, Albert Einstein combined these existing conceptual and mathematical formulations into an integrated and a unified approach, without giving reference to these contributions. Herman Minkowski formulated the relativity theory in terms of a four-dimensional-vector-field formulation. On the technical side, the paper shall focus on the infinities appearing in the Lorentz and the Poincaré transformations of energy and momentum, which cause problems in the formulation of field theories. An attempt has been made to avoid these infinities by scaling of coördinates.
**Keywords:** Relativity, quantum field theory, infinities, singularities.

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## EXACT SOLUTIONS FOR THE FLOW OF A GENERALIZED OLDROYD-$B$ FLUID INDUCED BY A CONSTANTLY ACCELERATING PLATE BETWEEN TWO SIDE WALLS PERPENDICULAR TO THE PLATE

**M. Kamran**
*Abdus Salam School of Mathematical Sciences,*
*Lahore-Pakistan.*

The unsteady flow of an incompressible generalized Oldroyd-$B$ fluid induced by a constantly accelerating plate between two side walls perpendicular to the plate has been studied using Fourier sine and Laplace transforms. The obtained solutions for the velocity field and shear stresses, written in terms of the generalized $G$ and $R$ functions, are presented as sum of the similar Newtonian solutions and the corresponding non- Newtonian contributions. For $\alpha = \beta = 1$ and $\lambda_r \to \lambda$ these solutions are going to the corresponding Newtonian solutions. Furthermore, the solutions for generalized Maxwell fluids as well as those for ordinary Oldroyd-$B$ and Maxwell fluids, performing the same motion, are also obtained as limiting cases of our general solutions. In the absence of the side walls, namely when the distance between the two walls tends to infinity, the solutions corresponding to the motion over an infinite constantly accelerating plate are recovered. For $\lambda_r \to 0$ and $\beta \to 1$, these solutions reduce to the known solutions from the literature. Finally, the effect of the material parameters on the velocity profile is spotlighted by means of the graphical illustrations.

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## CLASSES OF MEASURES GENERATED BY CAPACITIES

**G. E. Karadzhov**
*Institute of Mathematics and Informatics,*
*Bulgarian Academy of Sciences,*
*Sofia-Bulgaria.*

We introduce classes of measures in the half-space $\mathbf{R}^{n+1}_{+}$ generated by Riesz, or Bessel, or Besov capacities in $\mathbf{R}^n$, and give a geometric charac- terization as Carleson-type measures.

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## BOUNDS ON THE LARGEST FAMILIES OF SUBSETS WITH FORBIDDEN SUBPOSETS

**Gyula O. H. Katona**
*Alfred Rényi Institute of Mathematics,*
*Hungarian Academy of Sciences,*
*Budapest-Hungary.*

Let $[n] = 1, 2, \ldots, n$ be a finite set, families $\mathcal{F}$ of its subsets will be investigated. An old theorem of Sperner (1928) says that if there is no inclusion ($F \in \mathcal{F}, G \in \mathcal{F}, F \neq G$ then $F \not\subset G$ then the largest family under this condition is the one contaning all $\lfloor \frac{n}{2} \rfloor$-element subsets of $[n]$. This theorem has many consequences. It helps to find (among others) the maximum number of mimimal keys in a database, the maximum number of subsums of secret numerical data what can be released without telling any one of the data, bounds of the distribution of the sums $\sum \pm a_i$ for the vectors $a_i (1 \leq i \leq n)$.
We will consider its certain generalisations in the present lecture. They are useful in proving theorems in number theory. geometry, etc. Again, the maximum size of $\mathcal{F}$ is to be found under the condition that a certain configuration is excluded. The configuration here is always described by inclusions. More formally, let $P$ be a poset. The maximum size of a family $\mathcal{F} \subset 2^{[n]}$ which does not contain $P$ as a (non-necessarily induced) subposet is denoted by $La(n, P)$.
If $P$ consist of two comparable elements, then Sperners theorem gives the answer, the maximum is $\binom{n}{\lfloor \frac{n}{2} \rfloor}$.
In most cases, however $La(n, P)$ is only asymptotically determined in the sense that the main term is the size of the largest level (sets of size $\lfloor \frac{n}{2} \rfloor$) while the second term is $c/n$ times the second largest level where the lower and upper estimates contain different constants $c$.
Let e.g. the poset $N$ consist of 4 elements illustrated here with 4 distinct sets satisfying $A \subset B, C \subset B, C \subset D$. In a relatively new paper the author jointly with J.R. Griggs determined $La(n, N)$.
**Theorem:**
$\binom{n}{\lfloor \frac{n}{2} \rfloor}(1 + \frac{1}{n} + \Omega(\frac{1}{n^2})) \leq La(n, N) \leq \binom{n}{\lfloor \frac{n}{2} \rfloor}(1 + \frac{2}{n} + \mathcal{O}(\frac{1}{n^2}))$.
Similar results will be surveyed, also introducing a method.

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## ALMOST-SCHREIER DOMAINS

**Waseem Khalid**
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*Lahore-Pakistan.*

Joint work with: Tiberiu Dumitrescu

As an extension of the class of (pre)-Schreier domains introduced by P. M. Cohn and M. Zafarullah, we study a class of integral domains $D$ characterized by the property that whenever $a, b_1, b_2 \in D - \{0\}$ and $a$ divides $b_1 b_2$, there exist an integer k$\geq$ 1 and $a_1, a_2 \in D$ such that $a^k = a_1 a_2$ and $a_i$ divides $b_i^k$.
We call them almost-Schreier domains.Among other results,we show that an almost-Schreier domain has torsion t-class group and that a local one-dimensional Noetherian domain is almot-Schreier.

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## ON IMPROVEMENT OF JENSEN'S INEQUALITY AND APPLICATIONS ON MEAN

**Muhammad Adil Khan**
*Abdus Salam School of Mathematical Sciences,*
*Lahore-Pakistan.*

Joint work with: J. Pecaric

An improvement of Jensen's inequality for convex and monotone functions is given as well as various applications for mean. Similar results for related inequalities of Jensen's type are also obtained.

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## RAYLEIGHT TAYLOR INSTABILITY IN THE PRESENCE OF ROTATING COMPRESSIBLE FLUID THROUGH POROUS MEDIA

**Aiyub Khan**
*Department of Mathematics and Statistics,*
*Jai Naraian Vyas University,*
*Rajasthan-India.*

The hydromagnetic instability of a startified layer of viscous compressible rotating fluid through porous media in the presence of vertical magnetic field is considered. The solution has been obtained through the use of variational principle. The dispersion relation is derived for a layer having exponentially density startification along the vertical. It is found that viscosity has stabilizing influence while permeability of porous medium, magnetic resistivity, coriolis forces and compressibility have destabilizing influence on the growth rate of unstable mode of system.

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## AN APPLICATION OF INTUITIONISTIC FUZZY SOFT SETS IN FINANCIAL DIAGNOSIS OF FIRMS

**Muhammad Athar Kharal**
*Bahauddin Zakariya University,*
*Multan-Pakistan.*

Intutionistic fuzzy sets are an extension of the notion of fuzzy set. Soft sets is state-of-the-art approach to deal with incomplete knowledge within information systems. Intuitionistic fuzzy soft sets combine the strengths of both soft sets and intuitionistic fuzzy sets. The discipline of economic-financial diagnosis is heavily dependent upon value judgments of experts, qualitative variables and variables of relative quantification. Thus making this field of applicable mathematics suitable for tools taken from fuzzy logic and its extensions, which allow the qualitative variables to be properly treated and the subjective judgments to be modelled. In this paper, by using intutionsitic fuzzy soft sets, we present a method of diagnosis for causes of bad financial performance shown by firms in an economy.

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## DEFORMATIONS OF DETERMINANTAL SCHEMES IN PROJECTIVE SPACE

**Jan O. Kleppe**
*Faculty of Engineering,*
*Oslo University College,*
*Oslo-Norway.*

A scheme $X \subset \mathbf{P^n}$of codimension c is called determinantal if its homogeneous saturated ideal can be generated by the $r \times r$ minors of a homogeneous $p \times q$ matrix $f_{ij}$ with $c = (p - r + 1)(q - r + 1)$.
When $r = min(p, q)$ we say that $X$ is standard determinantal. Let Hilb($\mathbf{P^n}$) be the Hilbert scheme (resp. Hilbert scheme of constant postulation, i.e. constant Hilbert function) parametrizing closed subschemes of $\mathbf{P^n}$ of dimension $n - c > 0$ (resp. 0). Given integers $a_1 \leq a_2 \leq ... \leq a_p$ and $b_1 \leq ... \leq b_q$, we denote by $W^r_{p,q}(\underline{b}; \underline{a}) \subset Hilb(\mathbf{P^n})$ the stratum consisting of determinantal schemes as above where $f_{ij}$ are homogeneous polynomials of degrees $a_i - b_j$.
We consider the following problems:

(1) Determine the dimension of $W^r_{p,q}(\underline{b}; \underline{a})$ in terms of $a_i$ and $b_j$
(2) Determine whether the closure of $W^r_{p,q}(\underline{b}; \underline{a})$ is an irreducible component of $Hilb(\mathbf{P^n})$, and
(3) Determine when $Hilb(\mathbf{P^n})$ is generically smooth along $W^r_{p,q}(\underline{b}; \underline{a})$.

By successively deleting columns of the matrix associated to a determinantal $X$, we get a nest ("flag") of closed subschemes $X = X1 \subset X2 \subset ... \subset \mathbf{P^n}$. We prove our results by considering the smoothness of the Hilbert flag scheme of pairs, or of nests as above, and its natural projections into the usual Hilbert scheme.
In this approach we need to prove that certain $Ext^1-$groups vanish. We succeed quite well for standard (resp. Gorenstein codimension 4) determinantal schemes because, in this case, the codimension of $X_i$ in $X_{i+1}$ is 1 (resp. 2). In the computations the Eagon-Northcott and Buchsbaum-Rim complexes are systematically used, and the deformations of a regular section of certain Cohen-Macaulay modules of ranks 1 or 2 play an important role. Several of the results are obtained with coworkers (notably R.M. Miro-Roig, but also J. Migliore, U. Nagel and C. Peterson).

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## ALGEBRAIC GEOMETRY AND MULTIVARIABLE $KP$-HIERARCHY

**Herbert Kurke**
*Humboldt Universitaet,*
*Berlin-Germany.*

Joint work with: D. Osipov, A. Zheglov

In the 80-ties and 90-ties of the last century some remarkable connections between appearently disparate subjects where discovered (or partially rediscovered from earlier developements around 1900 and in the late 1920-ties) It is summarized in a correspondence between the following objects:
(1) Certain pairs of subspaces of infinite-dim.Grassmannians (of the locally lineare compact space of Laurent series in one variable) (Schur pairs)
(2) (geometric data) Integral projective curves equipped with some additional geometric data.
A subclass of these corresponds to commutative rings of germs of linear differential operators (1 variable).
The geometric data can be interpreted as "Spectral data" of the differential operators. The deformation theory of these data is gouverned by a system of differential eqations, which contains as special case the Kortrweg-deVries eqation (developt originally in order to describe whater waves observed in shallow channels).
Since then several attempts were and are made to establish a similar correspondence in the higher-dim. case (eg. algebraic surfaces and commuting 2-variable differential operators, and a 2-dim.version of the KP-Hierrrarchy). In the talk I will briefly descriibe the classical (!-diim.) case and outline results in higher dimensions.

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## GEOMETRY OF MONGE-AMPERE EQUATIONS: FROM LIE TO LYCHAGIN

**Alexei Kushner**
*Institute of Control Sciences of Russian Academy of Sciences, Astrakhan-Russia.*

I'll present a survey of results on classification of the Monge-Ampere equations. The problem of classification of such equations goes back to Sophus Lie. In 1978 Valentin Lychagin noted that the classical Monge-Ampere equations and its multi-dimensional analogues admit effective description in terms of differential forms on the space of 1-jets of smooth functions. The first advantage of this approach is a reduction of the order of the jet space: we use a simpler space of 1-jets $J^1M$ instead of the space of 2-jets $J^2M$ where Monge-Ampere equations should be "ad hoc", being second-order partial differential equations. I'll talk about results of Lychagin and his scientific school.

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## SPECTRAL INEQUALITIES AND THEIR APPLICATIONS

**Ari Laptev**
*Imperial College London, London, UK.*

The study of the relationship between geometry and spectral properties is one of the oldest subjects of human interest. Over 2,000 years ago the Pythagoreans had already discovered connections between the length of a string and the tone it produced. Nowadays, in most areas of physics and engineering, it is important to ascertain how shape determines the frequencies of vibrations and their distribution. The spectral analysis of differential operators plays a crucial role in the area of quantum mechanics. Many phenomena can be described in terms of the discrete and continuous spectrum of a linear operator where one is often interested in regimes where a certain parameter is either very large or very small. A mathematically rigorous analysis usually requires a careful bound on approximation errors. This is where spectral estimates play a central part in the proof. The celebrated inequalities by Lieb-Thirring, for example, are of importance in the theory of the stability of matter and in the turbulence theory.

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## ON LOG-CONVEXITY FOR MAJORIZATION

**Naveed Latif**
*Abdus Salam School of Mathematical Sciences, Lahore-Pakistan.*

Joint work with: Matloob Anwar, J. Pečarić

We give generalizations of majorization and related results using some log-convexity criteria. We also give new Cauchy means and prove monotonicity of these means.

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## CHARACTERIZATION OF MAPPING IN $\theta$-OPEN SET

**Raja Muhammad Latif**
*Department of Mathematics and Statistics,*
*King Fahad University of Petroleum and Minerals,*
*Dahran-Saudi Arabia.*

Velicko [1968] introduced the concepts of $\theta$ - closure and $\theta$ - interior operations. The collection of all $\theta$ - open sets in a topological space $X$ forms a topology on $X$. In this talk we introduce $\theta$ - irresolute, $\theta$- closed, pre - $\theta$ - open and pre - $\theta$ - closed mappings and investigate properties and characterizations of these new types of mappings and also explore further properties of the well-known notions of $\theta$ - continuous and $\theta$ - open mappings.

**2000 Mathematics Subject Classification:**Primary 54C05, 54C08, 54C10.
**Keywords and phrases:** Topological Space, $\theta$ - open set, $\theta$ - continuous function, $\theta$ - irresolute function, $\theta$ - open function, $\theta$ - closed function, pre - $\theta$ - open function, pre - $\theta$ - closed function.

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## CLASSIFICATION OF $P$-GROUPS. THE MODULAR ISOMORPHISM PROBLEM

**Olav Arnfinn Laudal**
*Institute of Mathematics,*
*University of Oslo,*
*Oslo-Norway.*

Let $G$ be a finite $p$-group, and let $A := \mathbf{F}_p G$ be the corresponding group algebra. The well-known Isomorphism Problem, in some circles known as the Isomorphism Conjecture, asks wether the group-algebra $A$ determines the group $G$. There are a long list of published papers on the subject, but until 2003 the only serious partial success was Quillens paper showing that $A$ determined the graded structure of $G$. I had, in 1976, tried out a method using deformation theory, but gave it up because of technical problems. At the end of the 90ties one of my students, Inger Christin Borge, took up the study, applying the newly developed non-commutative deformation theory, with its Massey-products. In a sense this was a generalization of Serres work, on Demushkin-groups, extending the obstructions allowed from cup-products to, what I have termed, immediately defined Massey products. She obtained a couple of interesting results, which renewed my interest in the Isomorphism Problem. Using the Brauer-Jennings-Zassenhaus $M$-series for the $p$-group $G$, we thought, for a while, that my results from 1976, coupled with the non-commutative deformation theory developed in the 90-ties, implied the Isomorphism Conjecture, to the affirmative.
It turned out that there was a serious gap in the proof, so the Isomorphism Problem is still there.
In this talk I shall sketch the obstruction theoretical method, mentioned above, and show how this method may be used to compute all p-groups with a given group algebra.
Of course, I still do not know if there are more than one!

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## YANG-MILLS BAR CONNECTIONS OVER COMPACT KÄHLER MANIFOLDS

**Hông Vân Lê**
*Institute of Mathematics,*
*Academy of Sciences of the Czech Republic,*
*Prague-Czech Republic.*
*&*
*Institute of Mathematics,*
*Hanoi-Vietnam.*

Let $M$ be a compact Hermitian manifold and $E$ a Hermitian vector bundle over $M$. The Koszul-Malgrange criterion asserts that $E$ carries a holomorphic structure, if and only if there is a unitary connection $A$ on $E$ such that the $(0,2)$-component $F_A^{0,2}$ of the curvature $F_A$ of $A$ vanishes. Thus we shall call a unitary connection $A$ satisfying the Koszul-Malgrange criterion a unitary holomorphic connection. We introduce a Yang-Mills bar equation as the Euler-Lagrangian equation for the Yang-Mills bar functional which is equal to the square of the $L_2$-norm of the $(0,2)$-component $F_A^{0,2}$ of a unitary connection $A$. This equation

has an advantage over the equation for a holomorphic connection because the latter one is overdetermined on complex vector bundles of higher dimension and the first one is elliptic modulo a degeneracy which is formally generated by the action of the complex gauge group. Solutions of the Yang-Mills bar equation are called Yang-Mills bar connections. Among Yang-Mills bar connections over compact Kähler manifolds there is a distinguish class of almost holomorphic connections which include unitary holomorphic connections. We show the existence of non-trivial almost holomorphic connections over torus. We show that any almost holomorphic connection on a Hermitian vector bundle over a compact Kähler manifold with positive Ricci curvature is in fact a holomorphic connection, and any Yang-Mill bar connection over a compact Kähler 4-manifold with positive Ricci curvature is in fact a holomorphic connection. We also prove the short time existence of the negative Yang-Mill bar gradient flow whose degeneracy degree is twice larger than the dimension of symmetries of the equation. We also prove a long time existence of a quasi gradient flow of the Yang-Mills bar functional and discuss implications of this existence and its relations to the Hodge theory.

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## RATIONAL CUSPIDAL CURVES, SURFACE SINGULARITIES AND RATIONAL HOMOLOGY SPHERES

**Ignacio Luengo**
*Faculty of Mathematics,*
*Universidad Complutense,*
*Madrid-Spain.*

Joint work with: J. Fernandez de Bobadilla, A. Melle, A. Nemethi

I will report on some progress on the classification of rational cuspidal curves (RCC) and related open problems. We use the theory of superisolated surface singularities to stablish a connection between normal surface singularities whose link is an homology sphere (QHS) and plane rational cuspidal curves We use this connection in one direction to give new properties of RCC that allows us to advance toward the classification of rational cuspidal curves and in the other direction to give counterexamples to several conjectures on QHS that relates topological and analytic invariant of the singularity like genus or the Seiberg-Witten invariant.

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## HOW TO DEFINE THE DUAL OF A HIGHER-DIMENSIONAL LINEAR SYSTEM?

**Vakhtang Lomadze**
*A. Razmadze Mathematical Institute,*
*of the Georgian Academy of Sciences,*
*Tbilisi-Georgia.*

The concept of homotopy will be adapted from homological algebra to ARMA-models. Then, using this concept, we shall extend a classical duality to linear dynamical systems in several time variables. We shall show that in higher dimensions as well controllability and observability are dual concepts.

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## DIFFERENTIAL INVARIANTS

**Valentin V. Lychagin**
*Institute of Mathematics and Statistics,*
*Universitetet i Tromsø,*
*Tromsø-Norway.*

In the lecture vaeious methods of computing differential invariants will be outlined. The role and applications of differential invariants in geometry, differential equations as well as in economics and control theory will be discussed.

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## SOME EXACT SOLUTIONS FOR THE ROTATIONAL FLOW OF A GENERALIZED SECOND GRADE FLUID BETWEEN TWO CIRCULAR CYLINDERS

**Amir Mahmood**
*Abdus Salam School of Mathematical Sciences,*
*Lahore-Pakistan.*

Joint work with: Saifullah, Georgiana Bolat

The velocity field and the associated tangential stress corresponding to flow of a generalized second grade fluid between two infinite coaxial circular cylinders, are determined by means of the Laplace and Hankel transforms. At time $t = 0$ the fluid is at rest and at $t = 0^+$ cylinders suddenly begin to rotate about their common axis with a constant angular acceleration. The solutions that have been obtained satisfy the governing differential equations and all imposed initial and boundary conditions. The similar solutions for a second grade fluid and Newtonian fluid are recovered from our general solutions. The influence of the

fractional coefficient on the velocity of the fluid is also analyzed by graphical illustrations.
**Keywords:** Generalized second grade fluid, velocity field, shear stress, fractional calculus, Hankel and Laplace transforms.

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## HAMILTONIAN PROPERTIES OF DIRECTED TOEPLITZ GRAPHS

**Shabnam Malik**
*Abdus Salam School of Mathematical Sciences,*
*Lahore-Pakistan.*

An $(n \times n)$ matrix $A(a_{ij})$ is called a Toeplitz matrix if it has constant values along all diagonals parallel to the main diagonal. A directed Toeplitz graph is a digraph with Toeplitz adjacency matrix. In this paper we discuss conditions for the existence of hamiltonian cycles in directed Toeplitz graphs.

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## POTENTIALS AND SINGULAR INTEGRALS IN MORREY SPACES

**Alexander Meskhi**
*A. Razmadze Mathematical Institute,*
*of the Georgian Academy of Sciences,*
*Tbilisi-Georgia.*

Boundedness of fractional and CalderónZygmund singular integral operators in Morrey spaces with nonstandard growth will be discussed. These spaces are defined on quasimetric measure spaces. Twoweight estimates for fractional integrals will also be presented.

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## THE PROBLEM OF CHOOSING LOSSES FUNCTION IN BAYESIAN PROBLEM OF MANY HYPOTHESES TESTING AND OPPORTUNITIES OF THEIR OVERCOMING

**Abdul Mueed**
*Abdus Salam School of Mathematical Sciences,*
*Lahore-Pakistan.*

Joint work with: K. J. Kachiashvili

The problem of choosing losses function in Bayesian problem of many hypotheses testing is considered. There is shown that the most widely used are linear ($L_1$) and quadratic ($L_2$) losses functions. The linear losses function is equivalent to the application of stepwise losses function at Bayesian approach. Some times it leads to the results corresponding to Neyman-Pearson's criterion, the goal of which is to maximize the power of criterion at given significant level of criterion. The quadratic losses function $L_2$ is the smooth alternative of $L_1$, which provide obtaining sensible decision rule. For any kind of losses function, the risk function, in Bayesian problem of many hypotheses testing, contains the mistakes of two types: mistakes of the first and the second types. Bayesian decision rule minimizes the total effect of these mistakes, though, finally, it is unknown the share of each of them in optimal (in sense of Bayesian criterion) value of risk function. At solution many important problems the results caused by different mistakes significantly differ to each other. Therefore it is necessary to guarantee limitation of rising the most undesirable from these mistakes and to minimize the mistakes of the second type. For solving these problems in the work are stated and solved conditional Bayesian tasks of testing many hypotheses.

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## COHEN-MACAULAY MONOMIAL IDEALS OF CODIMENSION 2

**Muhammad Naeem**
*Abdus Salam School of Mathematical Sciences,*
*Lahore-Pakistan.*

We give a structure theorem for Cohen-Macaulay monomial ideals of codimension 2, and describe all possible relation matrices of such ideals. In case that the ideal has a linear resolution, the relation matrices can be identified with the spanning trees of a connected chordal graph with the property that each distinct pair of maximal cliques of the graph has at most one vertex in common.
**Keywords:** Monomial Ideals, Taylor Complexes, Linear Resolutions, Chordal Graphs.

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## ON SOME SUBEXTENSIONS OF $k((X))/k$

**Asim Naseem**
*Abdus Salam School of Mathematical Sciences,*
*Lahore-Pakistan.*

Joint work with: Sever Angel Popescu

Let $K = k((X))$ be the field of Laurent series in one variable $X$, with coefficients in an arbitrary field $k$ of characteristic 0. Let $L$ be a subfield of $K$ containing $k \subset L \subset K$. We negate the possibility that: a Galois extension $K/L$ can be infinite. Equivalently, we prove that there doesn't exist infinite algebraic extension $K/L$ such that $L$ is Henselian. These answers in negative will hopefully make the study of intermediate subfields $L$ of $K$, easier. Some other nice results are given, by imposing some additional hypothesis on the field $k$.

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## STANLEY DECOMPOSITIONS AND LOCALIZATION


**Sumiya Nasir**
*Abdus Salam School of Mathematical Sciences,*
*Lahore-Pakistan.*



Let $K$ be a field and $I$ be a monomial ideal of the polynomial ring $S = K[x_1, \ldots, x_n]$ in n variables over a field $K$. We discuss the behavior of Prime filtration and Stanley decomposition of the multi graded module $S/I$ under the operation of localization with respect to the variable.


**Keywords:** Monomial Ideals, Prime Filtrations, Stanley decompositions, Stanley Ideals.


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## HEYTING ALGEBRA AND ITS DUAL


**Muhammad Nawaz**
*Balochistan University of Information Technology,*
*Engineering and Management Sciences,*
*Quetta-Pakistan.*



We define a Heyting algebra as a lattice $A$ equipped with a binary operation $h$ defined as $c \leq h(a, b)$ iff $inf(c, a) \leq b$ and its dual as a lattice $A$ equipped with a binary operation $g$ defined as $g(a, b) \leq c$ iff $b \leq sup(a, c)$. We prove that $A$ equipped with binary operations $g$ and $h$ is a Boolean algebra if and only if any one of following equivalent conditions is satisfied:
i. $sup(a, h(a, 0))1$

ii. $inf(a, g(a, 1))0$
iii. $g(a, 1)h(a, 0)$

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## NEW EXACT SOLUTIONS CORRESPONDING TO THE SECOND PROBLEM OF STOKES FOR SECOND GRADE FLUIDS

**M. Nazar**
*Abdus Salam School of Mathematical Sciences,*
*Lahore-Pakistan.*

Joint work with: Corina Fetecau, D. Vieru, C. Fetecaua

New exact solutions for the velocity field corresponding to the second problem of Stokes, for second grade fluids, have been established by the Laplace transform method. These solutions, presented as a sum of the steady-state and transient solutions, are in accordance with the previous solutions obtained by a different technique. The required time to get the steady-state is determined by graphical illustrations. This time decreases if the frequency of the velocity increases. The effects of the material parameters on the decay of the transients are also investigated by graphs.

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## GENERALIZED CONE METRIC SPACES AND FIXED POINT THEOREMS

**Talat Nazir**
*Lahore University of Management sciences,*
*Lahore-Pakistan.*

A notion of generalized cone metric space is introduced, and some convergence properties of a sequence are proved. Also some fixed point results for mapping satisfying certain contractive conditions are obtained. Our results complement, extend and unify several well known results in the literature.

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## POPOVICIU'S INEQUALITY FOR FUNCTIONS OF A VECTOR VARIABLE

**Constantin P. Niculescu**
*University of Craiova,*
*Craiova-Romania.*

Joint work with: M. Bencze and F. Popovici

T. Popoviciu [11] has proved in 1965 an interesting characterization of convex functions on intervals, relating the values at the barycenters of different subsets of a given finite set of points. In the simplest case his result reads as follows:
**Theorem** *Let $f : I \to \mathbb{R}$ be a continuous function. Then $f$ is convex if, and only if,*

$$\frac{f(x_1)+f(x_2)+f(x_3)}{3}+f\left(\frac{x_1+x_2+x_3}{3}\right)$$
$$\geq \frac{2}{3}\left[f(\frac{x_1+x_2}{2})+f(\frac{x_2+x_3}{2})+f(\frac{x_3+x_1}{2})\right]$$

*for all $x_1, x_2, x_3 \in I$.*
The aim of our talk is to develop a higher dimensional analogue of usual convexity based on this characterization.
**2000 Mathematics Subject Classification:** Primary 26A51, 26D15; Secondary 26B25.
**Key words and phrases:** Popoviciu.s inequality, convex function, convex combination.

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## MORE ON $F$-DERIVATIONS OF $BCI$-ALGEBRAS

**Farhat Nisar**
*Queen Mary College,*
*Lahore-Pakistan.*

In [14], the authors proved that if $X$ is a $BCI$-algebra then a self map $df$ defined as $df(x) = o*(o*f(x)) = fx$ is a $(l,r)$-$f$-derivation of $X$, where $f$ is an endomorphism of $X$. We generalize this result as a self map $df$ of a $BCI$-algebra $X$ defined as above is a $f$-derivation of $X$, for all $xX$, where $f$ is an endomorphism of $X$. We establish some properties related to this concept. We investigate that a self map $df$ of a $p$-semisimple $BCI$-algebra $X$ is a $(l,r)$-$f$-derivation if it satisfies the condition $df(x*y) = df(x)*f(y)$ instead of $df(x*y) = (df(x)*f(y))(f(x)*df(y))$, for all $x, yIX$, where $f$ is an endomorphism of $X$, as shown in [14] and it is a $(r,l) - f$-derivation if it satisfies the condition $df(x*y) = f(x)*df(y)$ instead of $df(x*y) = (f(x)*df(y)(df(x)*f(y))$, for all $x, yIX$, where $f$ is an endomorphism of X as given in [14]. We also characterize f-derivations of $BCI$-algebras.

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## RECURRENCE RELATION FOR JONES POLYNOMIAL

**A. R. Nizami**
*Abdus Salam School of Mathematical Sciences,*
*Lahore-Pakistan.*

We give a new method to compute Jones polynomials of braid links. The method is developed using simple recurrence relation. For large positive braids we obtain polynomials, not the general Laurent polynomials. The recurrence relation helps in proving many qualitative results. We also give a general expansion formula for Jones polynomials.

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## TAYLOR-COUETTE FLOW OF A GENERALIZED OLDROYD-B FLUID DUE TO A CIRCULAR CYLINDER SUBJECT TO A TIME-DEPENDENT COUPLE

**Shaikh Tajuddin Nizami**
*Department of Computer Science and Information Technology,*
*NED University of Engineering and Technology,*
*Karachi-Pakistan.*

Joint work with: Nasiruddin Khan

What happen if $n$ be $1/2$ in $\frac{d^n y}{dx^n}$?. This question concerning the derivative of nonintegral order. The idea of fractional calculus is not new. During the last two decade, some mathematicians carried forward their deep investigation into the area of fractional calculus.
The fractional calculus has become an exciting new mathematical tool for the solution of diverse problems of mathematical sciences and engineering.

In this paper we compare some fractional order dynamical systems with classical dynamical systems in control theory using relation between fuzzy logic and fractional derivatives and realize that fractional order controllers have more flexibility to achieve controlling objectives.
**Keywords:** Fractional calculus, Fractional derivatives, Fuzzy Logic , Classical dynamical systems, Fractional order dynamical systems.

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## HOMOTOPY THEORY OF $C^*$-ALGEBRAS

**Paul Arne Østvær**
*Department of Mathematics,*
*University of Oslo,*
*Oslo-Norway.*

The talk will be an introduction to a newly constructed homotopy theory of $C^*-$algebras. We aim to explain the bridging of $C^*$-algebras and homotopy theory, and discuss some of the invariants arising in this theory.

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## BEYOND PLANARITY

**Janos Pach**
*École Polytechnique Fédérale de Lausanne (EPFL),*
*Lausanne-Switzerland.*

Many questions in discrete and computational geometry, graph drawing, polyhedral combinatorics boil down to well understood properties of planar graphs, discovered by Euler, Kuratowski, Appel and Haken, Lipton and Tarjan, and others a long time ago. However, if we slightly relax the notion of planarity, the classical methods often break down. For instance, we do not know the answer to the following simple question. We call a graph drawn in the plane quasiplanar if it has no three pairwise crossing edges. Does there exist a constant c such that the edges of any quasiplanar graph can be colored by c colors so that no two edges of the same color cross each other? In this talk, we give a short survey of generalizations of the Lipton-Tarjan separator theorem for planar graphs. In particular, we describe some new extensions obtained in joint work with Jacob Fox, which enable us to go "beyond planarity" and answer a number of similar questions for "almost" planar graphs.

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## THE SPECTRAL ANALYSIS OF SPACE-TIME MODELS

**Udjianna S. Pasaribu**
*Faculty of Mathematics and Natural Science,*
*Institut Teknologi Bandung,*
*Bandung-Indonesia.*

In 1974, Koopmans represented time series as a spectrum, by decomposing the series as a linear combination of sines and cosines. He also showed that for geophysical data the spectrum is more representative than time domain time series analysis.
This paper extends his idea to the space-time series, particularly for Generalized Space Time Autoregressive order one model, written as GSTAR $(1;1)$. The spectrum can be achieved by writing the model as linear combination of uncorrelated vector random series or as Vector Moving Average with infinite order or VMA $\infty$. The existence of VMA $\infty$ and the spectral density matrix are fulfilled by a proposition. For illustration, this paper also shows a simulation.
**Keywords:** Spectrum, GSTAR model, uncorrelated vector, VMA model.

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## ON HILBERT TYPE INEQUALITIES

**Josip Pečarić**
*Faculty of Textile Technology,*
*University of Zagreb,*
*Zagreb-Croatia.*

Joint work with: Ivan Perić, Predrag Vuković

In this talk we give some new generalizations of classical Hilbert's inequality. We extend it to a general case with $k \geq 2$ non-conjugate exponents. The established technique is then applied to the case where functions are defined on $\mathbb{R}^n$, and in the case of conjugate exponents and some special homogeneous kernels it is shown that the obtained inequalities are the best possible. A generalization of the Hilbert-Pachpatte inequality and refinements of the Hilbert inequality using the Laplace transform are also presented. The problem of the best possibility of the Hilbert inequality with a general homogeneous kernel is considered in the case where functions are defined on $\mathbb{R}_+$.

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## SOME OPEN PROBLEMS RELATED THE HILBERT TYPE INEQUALITIES

**Ivan Perić**
*Faculty of Food Technology and Biotechnology,*
*University of Zagreb,*
*Zagreb-Croatia.*

Although it originated at the beginning of the 20th century, Hilbert's inequality is still of great interest. The purpose of this lecture is to give some (bias) account of the recent results in this area. Some open problems in determination of the best possible constant in non-conjugate Hilbert type inequalities will be outlined.
**2000 Mathematics Subject Classification:** Primary 26D10, 26D15
**Keywords:** Hilbert type inequalities, non-conjugate exponents, the best possible constants, Hardy-Littlewood-Sobolev inequality.

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## 25 YEARS OF SINGULAR

**Gerhard Pfister**
*University of Kaiserslautern,*
*Kaiserslautern-Germany.*

The computer algebra system SINGULAR has its 25th birthday. The talk will explain the interplay between computer algebra and its applications and illustrate the development during the last 25 years. We will see that needs in algebraic geometry and singularity theory always forced the development of the system. Deep theorems in group theory could be solved using the system. New applications to electrical engineering will be explained. We will also see applications to coding theory, partial differential equations. SINGULAR can also solve a Sudoku.

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## ON SOME METRIC INVARIANTS IN $p$-ADIC FIELDS

**Sever Angel Popescu**
*Technical University of Civil Engineering,*
*Bucharest-Romania.*

In this paper we introduce some metric invariants for an algebraic element of a fixed $p$-adic field $K$. We extend the definition for a transcendental element $T$ of the complex $p$-adic field. We apply this theory to generalize the known Sen-Ax inequality. We also answer to two open problems from a paper of A. Popescu, N. Popescu, M. Vajaitu and A. Zaharescu (Acta Arithmetica, 103.1 (2002)).

## A TWISTED WEYL CALCULUS ASSOCIATED TO QUANTUM SYSTEMS IN MAGNETIC FIELDS

**Radu Purice**
*Institute of Mathematics of Romania Academy,*
*Bucharest, Romania.*

In the presence of a variable magnetic field, the Weyl pseudo differential calculus must be modified. The usual modification, based on "the minimal coupling principle"' at the level of the classical symbols, does not lead to gauge invariant formulae if the magnetic field is not constant. We associate to a given magnetic field a perturbation of the canonical symplectic structure on the phase space and define a gauge covariant quantization. The underlying symbolic calculus is a deformation, defined in terms of the magnetic flux through triangles, of the classical Moyal product. A structure of twisted crossed product is obtained on the algebra of observables.

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## UNIFORM HAMILTONICITY IN CUBIC 3-CONNECTED $k$-HALIN GRAPHS

**Ahmad Mahmood Qureshi**
*Abdus Salam School of Mathematical Sciences,*
*Lahore-Pakistan.*

Let $H = T \cup C$ be a Halin graph, $T$ being a tree and $C$ the outer cycle. A $k$-Halin graph $G$ can be obtained from $H$ by adding edges while keeping planarity, joining vertices of $H - C$, such that $G - C$ has at most $k$ cycles. We prove that, in the class of cubic 3-connected graphs, all 7-Halin graphs are 1-edge hamiltonian while all 6-Halin graphs are uniform hamiltonian. These results are best possible.
**Keywords:** Halin graph, $k$-Halin graph, hamiltonian, 1-edge hamiltonian, uniform hamiltonian.

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## A CLASS OF PINCHED DOMAINS

**Shafiq ur Rahman**
*Abdus Salam School of Mathematical Sciences,*
*Lahore-Pakistan.*

Joint work with: Tiberiu Dumitrescu

We describe the non-Dedekind integral domains $D$ having an ideal $Q$ such that every non-invertible ideal of $D$ can be written as $JQ$ for some invertible ideal $J$. A star operation analog of this property is also studied.

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## DEPTH, STANLEY DEPTH AND DIMENSION OF MULTIGRADED MODULES

**Asia Rauf**
*Abdus Salam School of Mathematical Sciences,*
*Lahore-Pakistan.*

Let $K$ be a field and $S = K[x_1, \ldots, x_n]$ a polynomial ring in $n$ variables over a field $K$. We discuss the behavior of depth, Stanley depth and dimension along short exact sequences of finitely generated multigraded $S$-modules and under reduction modulo an element. We also study the behavior of depth, Stanley depth and dimension on algebra tensor product.

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## ENERGY MNIMIZATION RELATED TO NONLINEAR SCHRODINGER EQUATION

**Nauman Raza**
*University of the Punjab,*
*Lahore-Pakistan.*

In this paper, we study the applications of the Sobolev gradient technique to the problem of minimizing a Schrodinger functional associated with a nonlinear Schrodinger equation. We show that gradients in a suitably chosen Sobolev space (Sobolev gradients) can be used in finite difference and finite element settings in a computationally efficient way to find minimum energy states of Schrodinger functionals.

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## CAUCHY MEANS INVOLVING CHEBYSHEV FUNCTIONAL

**Atiq ur Rehman**
*Abdus Salam School of Mathematical Sciences,*
*Lahore-Pakistan.*

Joint work with: J. Pečarić

We give logarithmic convexity for Chebyshev functional as well as some related mean value theorems of Cauchy type. Also we give proofs of some mean value theorems related to generalized form of Chebyshev functional.

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## CONTINUITY ESTIMATES WITH RESPECT TO VOLATILITY FOR THE AMERICAN FOREIGN EXCHANGE OPTION

**Nasir Rehman**
*Abdus Salam School of Mathematical Sciences,*
*Lahore-Pakistan.*

Joint work with: Malkhaz Shashiashvili

We consider the American foreign exchange put option in one-dimensional diffusion model for the exchange rate process. The volatility is assumed to be arbitrary strictly positive bounded function of time. We derive several continuity estimates for the American foreign exchange put value process, optimal hedging portfolio and the corresponding consumption process with respect to volatility function. To establish the latter estimates we rely essentially on the crucial inequalities for the Snell envelopes and their components.

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## A STATIONARY TERNARY $C^4$ SCHEME FOR CURVE SKETCHING

**Kashif Rehan**
*University of the Punjab,*
*Lahore-Pakistan.*

Joint work with: Shahid S. Siddiqi

A new stationary ternary 5-point approximating subdivision scheme is presented which generates $C^4$ limiting curve and its limiting function has a support on $[-5, 4]$. The analysis of the scheme is shown using Laurent polynomial method and three examples illustrated the usefulness of the scheme.

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# COUNTEREXAMPLES TO ALEXANDER GROTHENDIECK'S PROBLEMS

**Oleg I. Reinov**
*Institute of Mathematics and Mechanics,*
*St. Petersburg State University,*
*St. Petersburg-Russia.*

In 1932, Stefan Banach posed his famous problem on the existence of a (Schauder) basis in any separable Banach space (the first example of a concrete basis in non-hilbertian space, namely, in $C[0,1]$ was given by Schauder). Since that time many mathematicians tried to find and did it for most classical Banach spaces (A. Haar, J. Schauder, J. Marcinkiewicz, and others). But the general Problem was still unsolved. For a long time, there were even no thought how to deal with this very important Problem. The positive solution would yield a lot of applications, would led to positive solutions of most of well known open problems in the corresponding fields.

In 1955, there appeared the famous fundamental work of Alexander Grothendieck "Produits tensoriels topologiques et espaces nucléaires" in Mem. Amer. Math. Soc. 16 (1955). In this treatise, A. Grothendieck has introduced and investigated a lot of new notions, especially, the notions of different approximation properties of spaces and operators (so-called AP and BAP; they generalize the property of a Banach space to have a basis). One of the nice consequence: if $X$ has the AP then every compact operator in $X$ in the closure of the set of all finite rank operators.

In his great work, A. Grothendieck has posed (in the very end of the treatise) more than 15 main unsolved Problems. Among them Problems: 1) of existence a Banach space without AP or BAP; 2) of existence a space with AP, but without BAP; 3) of equivalence of AP and BAP for all spaces. Concerning the last question, he had showed that for Banach spaces with weakly compact identity maps (for reflexive spaces) these two properties are the same. This led him to ask: 4) whether any weakly compact operator with AP possesses also BAP?

All the problems mentioned above were later solved in negative. As to the question on weakly compact operators, it it a paper of mine, – "Un contre-exemple a une conjecture de A.Grothendieck" (in C. R. Acad. Sc. Paris, Serie I.Vol. 296, 1983), – where the first counterexample for this problem was constructed.

In our lecture we shall discuss Grothendieck's approximation properties from different points of view. In particular, we are going to bring knew striking (counter) examples to "weakly compact" problem of A. Grothendieck. In this small survey, we will concern also some results of A. Grothendieck of 1955, which were shown by me to be invalid.

## ONE AND TWO WEIGHT PROBLEMS FOR ONE SIDED OPERATORS IN $L^{p(x)}$ SPACES

**M. Sarwar**
*Abdus Salam School of Mathematical Sciences,*
*Lahore-Pakistan.*

Joint work with: A. Meskhi

Criteria for the boundedness of one-sided maximal and fractional operators in weighted Lebesgue spaces with variable exponent have been established. In particular, we derived:

(1) one-weight inequality for one-sided maximal operators;
(2) two-weight estimates (criteria) for one-sided fractional maximal operators;
(3) Fefferman-Stein type inequality for one-sided fractional maximal functions;
(4) the trace inequality for one-sided potentials;
(5) a generalization of the Hardy-Littlewood theorem for the Riemann- Liouville and Weyl transforms.

All these results are obtain for general type weights.
The boundedness of one-side operators in unweighted Lebesgue spaces was established in [1].
The talk is based on the joint work with V. Kokilashvili and was announced in [2].

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## THE STRUCTURE OF ENDOMORPHISM RINGS OF LOCAL COHOMOLOGY MODULES AND APPLICATIONS TO CONNECTEDNESS

**Peter Schenzel**
*Martin-Luther-Universität Halle-Wittenberg,*
*Institut für Informatik,*
*Halle (Saale), Germany.*

Let $(R, m)$ denote a local Noetherian ring. For an ideal $I \subset R$ let $H^i_I(R), i \in \mathbb{Z}$, denote the local cohomology modules of $R$ with respect to $I$. The main aim of this talk is to study the endomrophism rings of certain local cohomology modules $H^i_I(R)$, in particular when $R$ is a Gorenstein ring and $i = grade\ I$.
Let $(R, m)$ denote a complete local Gorenstein ring with $\dim R = n$. Suppose that $I$ is an ideal such that $R/I$ has an isolated singularity in $\{m\}$. Then we show that the endomorphism ring of $H^c_I(R), c = grade\ I \geq 2$, is canonically isomorphic to $R$ if and only if $H^{n-1}_I(R) = H^n_I(R) = 0$.
Moreover we study the indecomposibility of local cohomology modules, the connectedness of $Spec\, R \setminus V(I)$, and the Bass numbers of certain local cohomology modules. It shown that the local cohomology module $H^c_I(R), c = height\ I$, is indecomposable if and only if $V(I_d)$ is connected in codimension one. Among others we show also that the endomorphism ring of $H^c_I(R)$ is a local Noetherian ring if $dim\, R/I = 1$.

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## GAUSS MANIN-CONNECTIONS AND BRIESKORN MODULES

**Khurram Shabbir**
*GC University,*
*Lahore-Pakistan.*

Let $f \in C[X_1, X_2, \ldots, X_n]$ be the homogeneous polynomial and $M(f)$ be the corresponding Milnor. This algebra $M(f)$ is a graded $C$-algebra and its dimension is Milnor number denoted as $\mu(f)$ when $f$ has an isolated singularity at the origin. Let $B(f)$ be the Brieskorn. It is known that $B(f)$ is a free $C[t]$ module of rank $\mu(f)$ when $f$ has an isolated singularity at the origin. The main result detects the torsion classes in the Brieskorn module, using explicit computations with differential forms.

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## A METHOD TO GENERATE LARGE CLASSES OF EDGE-ANTIMAGIC

**Muhammad Kashif Shafiq**
*Abdus Salam School of Mathematical Sciences,*
*Lahore-Pakistan.*

Joint work with: Martin Bača, Andrea Semaničová-Feňovčíková

A $(p, q)$-graph $G$ is said to be graceful if the vertices can be assigned the labels $\{1, 2, \ldots, q + 1\}$ such that the absolute value of the difference in vertex labels between adjacent vertices generate the set $\{1, 2, \ldots, q\}$. An $(a, d)$-edge-antimagic total labeling on a $(p, q)$-graph is defined as a one-to- one map taking the vertices

and the edges onto the integers $1, 2, \ldots, p+q$ with the property that the edge-weights (sums of endpoint labels and the edge label) form an arithmetic sequence starting from a and having a common difference $d$.
In this paper we use the connection between graceful labelings and edge-antimagic labelings for generating large classes of edge-antimagic total trees from smaller graceful trees.
**Keywords:** Graceful labeling, $\alpha$-labeling, edge-antimagic total labeling, tree.

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## CONVEX DATA MODELLING USING RATIONAL BI-QUARTIC FUNCTION

**Tahira Sumbal Shaikh**
*University of the Punjab,*
*Lahore-Pakistan.*

The Rational bi-quartic partially blended function [1], an extension of rational quartic function [2] is used for convex data modelling. The rational bi-quartic partially blended function has eight free parameters in each rectangular patch. Simple sufficient data dependent constraints are derived on these free parameters to create convex models for convex data. The presented scheme is local, computationally economical and verified numerically. The high degree of the rational function used in this paper, also make it suitable candidate for interactive CAD.

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## HYPERSURFACES DECOMPOSITION AND DE-RAHM COHOMOLOGY

**Hani Shaker**
*COMSATS Institute of Information Technology,*
*Lahore-Pakistan.*

Let $P$ be a polynomial in two variables with complex coefficients. The question under investigations is, how many irreducible components the plane curve $C : P(x, y) = 0$ has? The answer to this question is directly related to the study of the topology of the complement of $C$ in the complex plane $\mathbb{C}^2$ using de Rham cohomology.

The main problem is to extend this result for more variables and to obtain other related results on algebraic affine hypersurfaces.

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## SPHERICALLY SYMMETRIC GRAVITATIONAL COLLAPSE

**Muhammad Sharif**
*University of the Punjab,*
*Lahore-Pakistan.*

In this paper, we discuss gravitational collapse of spherically symmetric spacetimes. We derive a general formalism by taking two arbitrary spherically symmetric spacetimes with $g_{00} = 1$. Israel's junction conditions are used to develop this formalism. The formulae for extrinsic curvature tensor are obtained. The general form of the surface energy-momentum tensor depending on extrinsic curvature tensor components is derived. This leads us to the surface energy density and the tangential pressure. The formalism is applied to two known spherically symmetric spacetimes. The results obtained show the regions for the collapse and expansion of the shell.

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## DISCRETE TIME HEDGING OF THE AMERICAN OPTION

**M. Shashiashvili**
*Tbilisi State University,*
*Tbilisi-Georgia.*

Joint work with: S. Hussain

In a complete financial market we consider the discrete time hedging of the American option with a convex payoff. It is well-known that for the perfect hedging the writer of the option must trade continuously in time, which is impossible in practice. In reality, the writer hedges only at some discrete time instants.
The perfect hedging requires the knowledge of the partial derivative of the value function of the American option in the underlying asset, the explicit form of which is unknown in most cases of practical importance. Several approximation

methods have been developed for the calculation of the value function of the American option.
We claim in this article that having at hand any uniform approximation of the American option value function at equidistant discrete rebalancing times it is possible to construct a discrete time hedging portfolio, the value process of which uniformly approximates the value process of the continuous time perfect delta-hedging portfolio.
We are able to estimate the corresponding discrete time hedging error that leads to a complete justification of our hedging method for non-increasing convex pay-off functions including the important case of the American put. This method is essentially based on a new type square integral estimate for the derivative of an arbitrary convex function recently found by Shashiashvili (2005).

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## ON CYCLIC GROUPS ASSOCIATED WITH GENERIC MATRICES

**Arman Shokrollahi**
*University of Madras,*
*Tamil Nadu-India.*

A classical theorem of E. Noether asserts that if $R$ is a commutative ring, finitely generated over a field $k$, and $G$ is any finite group of $k$-automorphisms of $R$, then the fixed ring (or ring of invariants) $R^G$ is also finitely generated. The question naturally arises as to what extent Noether's theorem can be generalized to the noncommutative case. If $R$ is also Noetherian and $|G|^{-1} \in k$, all is well: $R^G$ is finitely generated [6]. However, it is false in general, even for $PI$ rings. Moreover, a result in Dicks and Formanek (and [4]), shows that almost the opposite of Noether's theorem holds in the free algebra. That is, they prove that if $G$ acts linearly on the free algebra $Fk\langle x_1, x_2, \cdots, x_d\rangle$, then $F^G$ is finitely generated if and only if $G$ acts by scalar matrices. For me details see [7,8,9,11, 10]. In the present paper we consider the analogous problem for an algebra of generic matrices. That is, let $Uk\{X_1, X_2, \cdots, X_d\}$ be the generic matrix algebra generated over a field $k$ by the $m \times m$ ($m \geq 2$) generic matrices $X_1, X_2, \cdots, X_d$ ($d \geq 2$). Let $G$ act linearly on $U$, that is, for each $g \in G$, $X_i^g \sum_j \alpha_{ij} X_j$, for $\alpha_{ij} \in k$. Thus $g$ corresponds to the $d \times d$ matrix $A(\alpha_{ij})$.

If $G$ consists of scalar matrices and $|G|^{-1} \in k$, then $U^G$ is always finitely generated. For, consider the free algebra $Fk\langle x_1, x_2, \cdots, x_d\rangle$ with the same action; since $U\overline{F}$, a homomorphic image of $F$, it follows that $U^G\overline{F}^G\overline{F^G}$ (since $|G|^{-1} \in k$), and thus $U^G$ is finitely generated since $F^G$ is finitely generated. However, the converse of this is false, as is shown by an example of Montgomery and Passman [5] of a nonscalar automorphism of order 3 of $2 \times 2$ generic matrices such that

$U^G$ is still finitely generated. Thus, the analog of Dick's and Formanek's theorem does not hold.

However, the main result of this paper shows, at least for cyclic groups, that for matrices which are large enough compared to $|G|$, the generic matrices behave like the free algebra. The following theorem is our theorem and the main result of this paper. (Theorem) Let $G\langle g\rangle$ be a cyclic group of order $n$ acting linearly on $Uk\{X_1, X_2, \cdots, X_d\}$, the generic matrix algebra of $m \times m$ matrices over the field $k$, where $d, m \geq 2$, and let $A$ be the matrix corresponding to $g$. Assume that $n$ is a unit in $k$ and that $A$ is not scalar. Then $U^G$ is not finitely generated whenever $m \geq n - [\sqrt{n}] + 1$.

Moreover, if $A$ has a characteristic root $\alpha$ such that $a^q 1$, some $q$ with 0.

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## A TERNARY THREE POINT SCHEME FOR CURVE DESIGNING

**Shahid S. Siddiqi**
*University of the Punjab,*
*Lahore-Pakistan.*

A stationary ternary 3-point approximating subdivision scheme is presented which generates $C^2$ limiting curve and its limiting function has a support on $[-3, 2]$. The analysis of the scheme is shown using Laurent polynomial method. Three examples are considered to show that this scheme behaves better than the scheme developed by Hassan and Dodgson.

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## MORSE THEORY OF A SET OF GROWING BALLS

**Dirk Siersma**
*Mathematisch Instituut,*
*Universiteit Utrecht,*
*The Netherlands.*

In this talk we consider as starting point a set of balls in n- dimensional Euclidean space. If the radius of those balls grows we will see that topology of the union of these balls is going to change. This is related to the topological Morse theory of a distance function.
We will consider balls of different radius $r_i$, centered at points $P_i$, together with the power distance

$$g_i(x) = \|xP_i\|^2 + r_i^2$$

It turns out that with the nearest distance criterium these functions define a teselation of n-space: known as the Power Diagram (generalizing Voronoi Diagrams).

These are well known subjects in computational geometry. The power diagram looks like a tropical hypersurface in tropical geometry and we will show they are indeed related.
Dual to the power diagram there is defined a Delaunay tesselation of the convex hull of the centers of the ball. Next we show that the Morse theory of the minimum of the distance functions $g_i$ can be discribed combinatoriallly and encoded by a discrete Morse function (following Forman) on this Delaunay tesselation.
At the end we give some examples. This is joint work with Martijn van Manen.

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## COHOMOLOGY OF CONFIGURATION SPACES OF COMPLEX PROJECTIVE SPACES

**Tanweer Sohail**
*Abdus Salam School of Mathematical Sciences,*
*Lahore-Pakistan.*

In this paper we compute topological invariants such as Betti numbers and cohomology algebra for some configuration spaces of complex projective spaces. To compute these invariants we shall use Kriz model and Serre-spectral sequences. We shall describe Sullivan models for these configuration spaces.

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## NEW APPROACHES TO THE HODGE CONJECTURE

**J. H. M. Steenbrink**
*Institute for Mathematics,*
*Astrophysics and Particle, Physics Radboud University*
*Nijmegen-The Netherlands.*

Let $X$ be a smooth complex projective variety. Each subvariety $Z$ of $X$ of codimension $p$ has a cohomology class $[Z]$ in $H^{2p}(X, \mathbb{C})$ which is a Hodge class, i.e. lies in

$$Hg^p(X) := H^{2p}(X, \mathbb{Q}) \cap H^{p,p}(X).$$

The famous Hodge Conjecture states that $Hg^p(X)$ is generated by cohomology classes of algebraic subvarieties.
The conjecture is not true for compact Kähler manifolds (Voisin) or with $\mathbb{Q}$ replaced by $\mathbb{Z}$ (Atiyah-Hirzebruch, Kollar). Hence possibly methods of projective geometry should be used. A relatively new approach, started by Thomas and Griffiths-Green and pursued by Brosnan, Pearlstein, Saito and others studies the variety $X$ by choosing some very ample line bundle $\mathcal{L}$ on $X$ and considering the family of hypersurface sections of $X$ given by sections of (powers of) $\mathcal{L}$. The main object of study is the group of admissible normal functions and their extensions

over the dual variety of $X$. We will review this work and reformulate it in terms of the Leray spectral sequence associated to a family of hypersurface sections of $X$.

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## UIR MATRIX ELEMENTS OF FINITE ROTATIONS OF $SO(2,1)$ DECOMPOSED ACCORDING TO THE SUBGROUP $T1$

**Ansaruddin Syed**
*Balochistan University of Information Technology, Engineering and Management Sciences, Quetta-Pakistan.*

Using a technique of Kalnins, UIR of principle series of $SO(2,1)$, decomposed according to the subgroup $T1$ , are realized on the space of homogeneous functions on the cone $0\,2\xi - 1\,2\xi - 2\,2\xi\,0$ as the carrier space. It is then shown that the matrix elements of an arbitrary finite rotation of $SO(2,1)$ are determined by those of two specific type of rotations, each depending on a single parameter; matrix elements of these two specific types of finite rotations are then explicitly computed.

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## SEXTIC SPLINE SOLUTION OF THIRD ORDER SINGULARLY PERTURBED BOUNDARY VALUE PROBLEM

**Ali Tabraiz**
*University of the Punjab, Lahore-Pakistan.*

Joint work with: Shahid S. Siddiqi

Various techniques are used to solve the singularly perturbed boundary value problems. Spline of degree six is used to determine the approximate solution of the following third order self adjoint singularly perturbed boundary value problem
$Ly = -\varepsilon y''' + p(x)y = f(x)$ , $p(x) \geq 0$
$y(0) = \alpha_0, y(1) = \alpha_1, y'(0) = \alpha_2$
or
$y(0) = \alpha_0, y(1) = \alpha_1, y''(0) = \alpha_2$
Three examples are considered for the numerical illustration of the method developed.

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## EFFECT OF THERMAL RADIATION ON MHD FREE CONVECTION HEAT AND MASS TRANSFER WITH DOUBLE DIFFUSION FROM VERTICAL SURFACES IN POROUS MEDIA

**Shyam Sunder Tak**
*Jai Naraian Vyas University,*
*Rajasthan-India.*

The effect of thermal radiation on heat and mass transfer characteristics of natural convection about a vertical surface embedded in saturated Darcian porous medium subjected to a magnetic field is studied taking into account the Soret and Dufour effects. The Rosseland approximation for the radiative heat flux is used in the energy equation. It is found that the similarity solution exists in the present case. The resulting set of coupled non-linear ordinary differential equations is solved numerically using shooting technique. Dimensionless velocity, temperature and concentration profiles are presented graphically for various values of involved parameters. Also, the Nusselt and Sherwood numbers are tabulated for different values of the involved parameters. It is found that the Nusselt number increases and Sherwood number decreases as the radiation parameter increases but both the Nusselt number and Sherwood number decrease as the magnetic field parameter increases.

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## COWSIK-NORI THEOREM FOR SQUAREFREE MONOMIAL IDEALS

**Naoki Terai**
*Saga University,*
*Saga-Japan.*

This report is based on a joint work with M. Crupi, G. Rinaldo and K. Yoshida. We discuss a generalization of Cowsik-Nori theorem for squarefree monomial idels. More precisely, we show the following two theorems:
**Theorem:** Let $S$ be a polynomial ring over a field. Let $I$ be a Stanley-Reisner ideal in $S$. Then the following conditions are equivalent:
(1)$S/I$ is a complete intersection.
(2)$S/I^i$ is Cohen-Macaulay for all $i \geq 1$.
(3)$S/I^i$ is Buchsbaum for all $i \geq 1$.
**Theorem:** Let $S$ be a polynomial ring over a field. Let $I$ be an edge ideal in $S$. Set $h = height\, I$. Then the following conditions are equivalent:
(1)$S/I$ is a complete intersection.
(2)$S/I^i$ is Cohen-Macaulay for some $i \geq h$.

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## D-MODULES, DERIVED CATEGORIES AND SINGULARITIES

**Lê Dung Trang**
*The Abdus Salam International Centre for Theoretical Physics,*
*Trieste, Italy.*

In this lecture we shall consider linear ordinary differential equations and we shall show that a natural way to study them is to consider solutions which define a complex which has two cohomologies, one in dimension 0 which represent "usual" holomorphic solutions, one in dimension 1 which represent distributions with support at points. This viewpoint generalizes to higher dimension for D-modules which are holonomic. There is a natural relation with singularities by considering vanishing cycles.

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## WEAKLY 1-COMPLETE SPACES

**Viorel Vajaitu**
*Department of Mathematics,*
*University of Lille,*
*Lille-France.*

In this talk I will discuss two basic notions in the function theory of several complex variables namely; weak 1-completeness and holomorphic convexity. Both are intimately related to the Levi problem in complex analysis.
The structure of the lecture is:
a) Weakly 1-complete spaces. Definitions and examples. Motivation of their study.
b) Criteris for holomorphic convexity of weakly 1-complete spaces.
c) Recent results and open questions.

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## SEMIDUALIZING MODULES AND BASS NUMBERS

**Sean Sather-Wagstaff**
*North Dakota State University,*
*Fargo-USA.*

Huneke has asked whether a local Cohen-Macaulay ring $R$ whose Bass numbers are eventually constant must be Gorenstein. We will show how the existence of a nontrivial semidualizing $R$-module implies that the Bass numbers of $R$ must grow at least linearly; moreover, the existence of a chain of $d+1$ semidualizing modules yield a degree-$d$ polynomial lower bound for the Bass numbers of $R$.

Furthermore, we show how information about the first few Bass numbers of $R$ yields information about the structure of the class of semidualizing $R$-modules.

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## SOME RESULTS ON $H$-POINTS IN $\mathbb{R}^2$

**Liping Yuan**
*College of Mathematics and Information Science,*
*Hebei Normal University,*
*Shijiazhuang-China.*
Joint work with: Penghao Cao

An $H$-point is a point of the set of corners of a tiling of $\mathbb{R}^2$ by regular hexagons. In this talk we discuss the number of H-points on any given line in $\mathbb{R}^2$, and we also prove a Minkowski-type theorem and a Blichfeldt-type theorem for $H$-points.

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## $l$-VOLTERRA QUADRATIC STOCHASTIC OPERATORS, LYAPUNOVE FUNCTIONS, TRAJECTORIES

**Akbar Zada**
*Abdus Salam School of Mathematical Sciences,*
*Lahore-Pakistan.*

We consider $l$-Volterra quadratic stochastic operators defined on $(m-1)$ dimensional simplex. Where l belongs to $\{1, 2, ..., m\}$. Under some conditions on coefficients of such operators we describe Lyapunov functions and apply them to obtain upper estimates for the set of limit points of the trajectories. We describe a set of fixed points of $l$-Volterra operators.

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## MORE THAN MOST IN CONVEXITY

**Tudor Zamfirescu**
*University of Dortmund,*
*Dortmund-Germany.*

In mathematics, one way of distinguishing between many and few is counting. For infinite sets this does not always work. Cardinal numbers can help. But even in Euclidean spaces sets can rarely be distinguished that way. Measure can help better. Or Baire categories. However these two ways to make a distinction may deliver different results! Porosity instead of rarity helps, by unifying the two results and increasing the degree of distinction.

We shall see how this can be further increased, thus helping to decide the outcome of a competition between smooth and strictly convex!

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## AN (IN) EQUALITY RELATED TO THE ONE-DIMENSIONAL HILBERT TRANSFORM

**R. L. Zeinstra**
*University of Hamburg,*
*Hamburg-Germany.*

In statistical quantum mechanics an important identity, known as the sum rule, states that the integral over the real axis of a certain spectral function, involving a positive function $u$ (characterizing inverse lifetime) and its usual Hilbert transform $\tilde{u}$, always has the value 1. It seems that the corresponding (mathematical) question, to determine the class of functions $u$ for which the identity is valid, first received real attention a few years ago, when Kondratjev and Alenitsyn considered the problem together with their students. They found that the spectral integral actually will be less than 1 in quite a few cases. Here we shall indicate that the value of the spectral integral is always $\leq 1$, and that the more interesting case of equality holds under a fairly natural condition to be imposed on the input function $u$. Previously, joint work in 2007 with Alenitsyn and Arshad had established equality for $u$ belonging to some subclass of $L^p$, $1 < p < \infty$. We also show that the class of admissable functions $u$ can be (further) widened by considering a slight modification of the spectral function.

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## TWO AND THREE DIMENSIONAL ARITHMETIC SEQUENCE

**Syed Qaiser Raza Zahedi**
*UNizam Pura, PO Kot Abdul Malik,*
*Lahore - Pakistan.*

Carl F. Gauss, the famous German mathematician gave world the theory of Arithmetic Sequence. That was more than two centuries ago from now, but now times have changed. With the beginning of the third millennium, we have come to the threshold of a new era and world of mathematics is more open than ever for researchers to do new explorations. With the help of Magic Squares, I have given the theory of Arithmetic Sequence a New Two Dimensional Concept. Magic Squares have quite an old history and origin in China which dates way back to 2800 BC. I observed that each Magic Square happens to be an Arithmetic Sequence beautifully placed on a simple plane of x, y co-ordinates.

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# EVOLUTION OF MATHEMATICS AS A TOOL

**Zeeshan**
*AKHSS Gilgit,*
*Gilgit - Pakistan.*

I will describe the evolution of math's parts in different eras.